MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 99 No. 5

MEETING OF 1939 MARCH 10

Professor H. C. Plummer, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Lawrence Alfred Cox, 47 Upper Green, Mitcham, Surrey ;
Henry Charles King, 238 Farnham Road, Slough, Bucks ;
Frederick Jones O'Connor, M.A., Trinity College, Dublin ; and
George William Reed, M.Sc., Royal Observatory, Cape of Good Hope, S. Africa.

The following candidates were proposed for election as Fellows of the Society, the name of the proposer from personal knowledge being appended :—

Robert Atkinson, R.N.R., Dene House, Coble Dene, North Shields (proposed by Wilfred Hall) ;
Herbert Henry John Barton, 48 Cambridge Drive, Lee, London, S.E. 12 (proposed by H. Spencer Jones) ; and
Joseph Jones, M.Sc., Headmaster, County School, Fishguard, Pembs. (proposed by T. E. R. Phillips).

One hundred and thirty-four presents were announced as having been received since the last ordinary meeting.

UNIVERSITY LIBRARY LEEDS

MEASURES OF THE RELATIVE SHIFTS OF THE LINE 5250·218 AND NEIGHBOURING LINES IN MT. WILSON SOLAR MAGNETIC FIELD SPECTRA

J. Evershed, F.R.S.

These spectra form part of those obtained in the year 1913 and used by Hale and his collaborators in their researches on the general magnetic field of the Sun *.

The spectra are 27 mm. wide, and divided into about 13 strips, each 2 mm. wide. The scale is 4·65 mm. to the angstrom unit, and the lines represent a section of the Sun along the central meridian in mid-latitudes.

The division into narrow strips is due to the compound quarter-wave plate which was fixed over the spectrograph slit, its function being to cut out in adjacent strips the red and violet components of the Zeeman doublets due to the magnetic field. As the general field, if it exists, is evidently very much weaker than in sunspots, the affected lines would not be split into doublets or triplets as in spot spectra, but would be more or less widened : hence the cutting out of the red and violet components in adjoining strips would have the effect of shifting the centre of an affected line towards red in one strip and towards violet in an adjoining strip. If the strips are numbered 1 to 13, the shift for the odd numbers would have one sign and the even numbers the opposite sign.

The problem of measuring the minute shifts to be expected is complicated by the presence of small irregular Doppler shifts, which I have shown to occur over the Sun's disc, especially between centre and limb, even at times of minimum solar activity †. There are, however, two criteria whereby the Zeeman effect may be recognised, for if the shifts on the whole tend to alternate plus and minus in adjoining strips they are likely to be Zeeman effects, whilst if the sign of the shifts changes in passing from the northern to the southern hemisphere that would also indicate the presence of a solar magnetic field. It happens that the spectra which Dr. Hale selected for me to measure in the year 1933 appeared to satisfy these two conditions fairly well, and so favoured the magnetic effect ‡. They were, however, obviously modified by small Doppler effects, or possibly by local magnetic fields.

The lines measured included the chromium line at 5247·6 and the iron line at 5250·6, but not the line at 5250·2, which, as I have shown in a recent paper §, gives a very much greater Zeeman effect than its near neighbour 5250·6. Here then we have a more certain means of distinguishing between Doppler and Zeeman effect in the relative shifts of these two iron lines, for they would be expected to have similar Doppler shifts, but very different Zeeman shifts.

* *Astrophysical Journal*, 47, 212.
† *M.N.*, 94, 96, 1933.
‡ *M.N.*, 94, 96, 1933.
§ *M.N.*, 99, 217, 1939.

I do not possess the spectra Hale selected for me to measure, or the measures themselves, as both were taken away by him ; but I have been able to measure nine other spectra he left with me, which include these lines.

A set of measures was made first by superposing an ordinary contact positive over a negative without reversing the plates end for end, but displacing the positive laterally by one 2-mm. interval, so that plus shifts in the negative might be covered by minus shifts in the positive. The four following lines were included in the measures :—

	5247·060 *Fe* 1	5247·576 *Cr* 2	5250·218 *Fe* 2	5250·656 *Fe* 3
Approximate widening in spots.	0·09	0·12	0·17	0·05

The widening here shown is the measured difference between the normal width of the lines and the width in spot spectra. The iron line at 5250·2 is clearly split into two in the penumbræ of spots, and is occasionally seen to have a central component in the umbræ.

Of the nine spectra measured, four did not satisfy the first condition—that is, the alternation of shifts in adjoining strips. The remaining five spectra were more favourable to the magnetic effect, although there were marked discrepancies. I give in Table I the mean values in angstroms of the residual shifts from these two sets of measures, disregarding the sign of the shifts.

TABLE I

	5247·0	5247·6	5250·2	5250·6
Mean of 4 spectra marked 1130 and 1176	0·0003	0·0002	0·0004	0·0004
Mean of 5 spectra marked 1176 and 1129	0·0011	0·0012	0·0012	0·0013
Mean of shifts exceeding 0·0015 A	0·0033	0·0036	0·0035	0·0039

The five spectra which seem to favour the Zeeman effect give mean shifts of about the same amount as found in my earlier measures ; but as regards the second condition, the change of sign of the shifts in going from the northern to the southern hemisphere, nothing can be said, as the requisite data are not with me. Latitudes are only marked on three spectra, and these are all in the same hemisphere. It is true that in each of these the strips of odd or even numbers have usually the same sign, as they should have if the shift is a Zeeman effect, so that if we disregard the relative shifts of the four lines the evidence favours a general magnetic field partly masked by the effects of motion. On the other hand, the relative shifts clearly refute the Zeeman effect and favour the Doppler interpretation, for all four lines give essentially the same shift, whereas they should follow the order of widening in sunspot spectra and of the magnetic effect in the laboratory, if due to a magnetic field.

In all of the nine spectra, shifts are found varying between 0·0015 A and 0·0050 A. In the third series of Table I are given the mean values of shifts exceeding 0·0015. Here again the values are almost the same in the different lines, 5250·6 showing, indeed, slightly larger shifts than 5250·2. These larger shifts, therefore, cannot be ascribed to local magnetic fields, but must certainly be due to motion.

Finally the three spectra marked 1129, which extend in solar latitude from $-25°$ to $-51°$, I have measured again with positives copied through the glass in a long-focus camera. The positive image is then superposed on the negative, film to film, but reversed end for end. In this way the positive image of any strip of spectrum falls on the corresponding strip of the negative, but the shifts are in opposite directions.

The position of the two lines having the greatest and least widening in spots in each of ten strips was measured and compared with the mean position, and the mean of each set disregarding the sign of the shift is given in Table II.

TABLE II

Plate 1129	No. of Strips measured	5250·2	5250·6
° °			
−27 to −31	10	0·0014	0·0020
−33 to −42	11	0·0018	0·0017
−43 to −51	10	0·0014	0·0012
	Means	0·0015	0·0016
Shifts exceeding 0·0015 *A in* 5250·2			
° °			
−27 to −31	4	0·0024	0·0032
−33 to −42	5	0·0030	0·0027
−43 to −51	4	0·0024	0·0019
	Means	0·0026	0·0026

Here again there is little or no evidence of a magnetic effect. The section $-43°$ to $-51°$ appears more favourable to the Zeeman effect than the other sections, but the difference of shift is too small to be of significance, for the Zeeman widening for these lines should be in the ratio three to one.

The method of reversing end for end has the advantage that it will detect any want of symmetry in a line, which might well be produced in the case of Zeeman triplets when one side component has been suppressed. In this case these two lines are perfectly symmetrical in these spectra.

Summary.—Nine of the Mt. Wilson solar magnetic field spectra obtained in the year 1913 have been measured by the positive on negative method, and the relative shifts of four lines determined. Two of these are subject to large Zeeman effects in sunspots and laboratory, whilst the other two are much less affected. The results clearly show that in these spectra the shifts observed are not due to a general magnetic field, or to local magnetic fields, but are Doppler effects due to motion, notwithstanding certain indications to the contrary.

THE URSA MAJOR CLUSTER

W. M. Smart

1. The community of motions of the five stars β, γ, δ, ϵ and ζ in Ursa Major was first noticed by Proctor about seventy years ago. About thirty years ago Hertzsprung * found eight more stars entitled to be considered members of the cluster, of which Sirius was the most noteworthy addition. Several calculations of the convergent point and of the parallaxes of the individual members of the cluster have been made by, amongst others, Hertzsprung *, Ludendorff †, Bottlinger ‡ and Rasmuson §; in the memoir by the last-named full details and references of previous investigations are given. The present investigation is based primarily on the proper motions of the *General Catalogue* recently published under the auspices of the Carnegie Institution of Washington.

Table I, List A, contains the relevant details of the 28 stars in Rasmuson's list, several of which were not definitely proved to be cluster-stars. According to the conclusions reached in paragraph 6 the five stars marked with an asterisk are not members of the cluster, and the two indicated by a double asterisk are doubtful members.

List B refers to 6 stars which were suggested by Plummer || for membership of the cluster. The single and double asterisks have the same meanings as for List A.

It will be noticed that the stars are scattered all over the sky, unlike the compact formation of the Taurus Cluster treated in a recent paper ¶.

2. The method adopted for the determination of the convergent point is as follows. Let θ be the position-angle of the total proper motion of a cluster-star; then

$$\tan \theta = \frac{\mu_\alpha \cos \delta}{\mu_\delta}. \tag{1}$$

Let $Q(a, d)$ denote that pole, in the northern hemisphere, of the great circle defined by θ. If (α, δ) are the co-ordinates of the star concerned, the values of a and d are given by

$$\cot (a - \alpha) = \sin \delta \tan \theta, \tag{2}$$

$$\sin d = \cos \delta \sin \theta. \tag{3}$$

Owing to the convention adopted with regard to d, we take θ in (3) to be

* *Ap. J.*, **30**, 1909.
† *Ast. Nach.*, **183**, 1909.
‡ *Ast. Nach.*, **198**, 1914.
§ *Lund Medd.*, Ser. II, Nr. 26, 1921.
|| *M.N.*, **76**, 134, 1916.
¶ *M.N.*, **99**, 168, 1939.

the position-angle, if the latter is between 0° and 180°, and the position-angle minus 180°, if the position-angle lies between 180° and 360°.

TABLE I

List A

No.	G.C.	Star	α (1950·0) h m s	δ (1950·0) ° ′	$\mu_\alpha \cos\delta$ (Unit: 0″·001)	μ_δ (Unit: 0″·001)	P.E. $\mu_\alpha \cos\delta$ (Unit: 0″·0001)	P.E. μ_δ (Unit: 0″·0001)
1*	1383	80 Psc	1 5 47	+ 5 23	−267	− 176	± 18	± 10
2*	2247	...	1 48 58	−50 27	− 61	− 6	38	35
3	2933	...	2 23 38	−15 34	− 61	− 43	58	31
4	5327	ξ Eri	4 21 11	− 3 52	− 49	− 57	31	19
5*	6274	β Eri	5 5 23	− 5 9	− 93	− 79	15	11
6	7543	β Aur	5 55 52	+44 57	− 51	− 4	8	7
7	8833	α CMa	6 42 57	−16 39	−537	−1210	8	7
8**	12532	α Vol	9 1 40	−66 12	− 2	− 104	15	24
9	14527	37 UMa	10 31 57	+57 20	+ 66	+ 33	10	11
10	15145	β UMa	10 58 50	+56 39	+ 82	+ 29	7	8
11	15438	δ Leo	11 11 27	+20 48	+146	− 138	10	6
12	16268	γ UMa	11 51 13	+53 58	+ 94	+ 4	5	6
13	16736	δ UMa	12 12 58	+57 19	+106	+ 3	6	8
14	17502	41 Vir	12 51 19	+12 41	+ 50	− 32	34	20
15	17518	ϵ UMa	12 51 50	+56 14	+113	− 11	7	8
16	17664	78 UMa	12 58 35	+56 38	+114	− 16	16	16
17	18133	ζ_1 UMa	13 21 55	+55 11	+124	− 28	7	8
18	18134	ζ_2 UMa	13 21 56	+55 11	+118	− 36	15	17
19	18155	80 UMa	13 23 14	+55 15	+118	− 24	11	13
20	18335	78 Vir	13 31 36	+ 3 55	+ 39	− 31	27	16
21*	19777	ζ Boo	14 38 46	+13 56	+ 52	− 26	16	10
22*	19991	ξ Boo	14 49 5	+19 18	+134	− 107	18	15
23	20342	45 Boo	15 5 6	+25 4	+184	− 177	22	14
24	20947	α CrB	15 32 34	+26 53	+119	− 98	8	6
25	21194	β Ser	15 43 53	+15 35	+ 66	− 55	13	10
26**	29428	...	21 2 0	+41 26	− 2	− 54	31	24
27	30376	76 Cyg	21 39 33	+40 39	− 15	− 45	26	17
28	32917	λ Psc	23 39 30	+ 1 30	−135	− 146	19	11
		List B						
29**	2212	χ Ceti	1 47 8	−10 56	−154	− 84	14	14
30	7197	P.G.C. 1420	5 42 23	−22 28	−287	− 371	13	14
31*	9012	P.G.C. 1768	6 50 32	+38 34	− 7	− 30	31	32
32**	12761	18 UMa	9 12 36	+54 14	+ 56	+ 55	15	13
33**	15652	P.G.C. 2999	11 21 19	+10 48	+169	− 81	9	10
34	17404	Gr 1930	12 46 29	+60 36	+107	− 5	35	25

Table II contains the values of a and d for the stars of Table I, List A. (The stars in List B were not used in this connection.)

The co-ordinates of the convergent point are then found from the consideration that the convergent point is one of the two poles of the great circle defined by the poles, Q, of the stars used in the calculations. Let (A, D) be the position of the convergent point, and let

$$x = \cos A \cot D, \tag{4}$$

$$y = \sin A \cot D. \tag{5}$$

TABLE II

Poles of the Great Circles

No.	a	d	l	m	n	Weight
	°	°				
1*	278·4	56·2	0·081	−0·550	0·831	2
2*	20·0	39·3	727	264	634	1
3	326·8	52·0	515	− 338	788	1
4	338·6	40·6	707	− 276	651	1
5*	352·4	48·8	653	− 087	752	2
6	275·3	44·9	065	− 706	706	3
7	18·0	22·9	876	285	389	3
8*	46·4	0·5	690	724	008	1
9	8·7	28·9	866	132	483	2
10	7·6	31·2	847	114	518	3
11	278·5	42·8	108	− 726	679	2
12	0·7	36·0	809	011	588	3
13	5·2	32·7	839	076	540	3
14	301·8	55·3	300	− 485	822	1
15	6·3	33·6	828	091	553	3
16	5·1	33·0	836	075	545	2
17	5·1	33·8	828	074	557	3
18	0·1	33·1	838	002	546	2
19	6·9	34·0	823	099	559	2
20	297·8	51·3	292	− 552	781	1
21*	335·4	60·2	451	− 206	868	2
22*	334·8	47·5	610	− 288	737	2
23	340·0	40·8	712	− 258	653	1
24	351·9	43·5	718	− 102	688	3
25	343·8	47·7	646	− 187	740	2
26*	224·1	1·6	− 718	− 696	027	1
27	222·6	13·9	− 715	− 657	240	1
28	263·5	42·7	−0·083	−0·729	0·679	2

Then we have * x and y given by

$$lx + my + n = 0, \tag{6}$$

in which

$$l = \cos a \cos d, \qquad m = \sin a \sin d, \qquad n = \sin d. \tag{7}$$

The values of l, m and n are given in Table II for each of the 28 stars. Equation (6) is the equation of condition, from a least-squares solution of

* See, for example, the author's *Stellar Dynamics*, 285, 1938.

which the values of x and y are determined. The co-ordinates A and D are then found by means of (4) and (5).

With A and D now known we can calculate for each star the position-angle of the total proper motion if the latter were exactly directed towards the convergent point (A, D). This is done for stars in both List A and List B.

The second columns of Table III contain the observed position-angles, and the third columns contain the corresponding calculated values. The weights in the fifth columns are the weights arbitrarily assigned to the individual stars in forming the normal equations derived from (6); they are based principally on the probable errors of the proper-motion components as given in Table I.

TABLE III

Position-Angles of the Proper Motions

List A

No.	Position-Angle			Wt.	No.	Position-Angle			Wt.
	Obs.	Cal.	Obs. – Cal.			Obs.	Cal.	Obs. – Cal.	
	°	°	°			°	°	°	
1*	236·6	227·7	+ 8·9	2	15	95·6	97·0	− 1·4	3
2*	264·4	248·6	+15·8	1	16	98·0	97·8	+ 0·2	2
3	234·8	232·0	+ 2·8	1	17	102·7	103·4	− 0·7	3
4	220·7	226·2	− 5·5	1	18	107·0	103·4	+ 3·6	2
5*	229·6	221·7	+ 7·9	2	19	101·5	103·6	− 2·1	2
6	265·5	269·2	− 3·7	3	20	128·5	129·6	− 1·1	1
7	203·9	203·4	+ 0·5	3	21*	116·6	128·9	−12·3	2
8*	181·1	173·6	+ 7·5	1	22*	128·6	128·9	− 0·3	2
9	63·4	60·9	+ 2·5	2	23	133·9	129·3	+ 4·6	1
10	70·5	70·5	0·0	3	24	129·5	131·6	− 2·1	3
11	133·4	127·4	+ 6·0	2	25	129·8	132·7	− 2·9	2
12	87·6	87·5	+ 0·1	3	26*	182·1	186·3	− 4·2	1
13	88·4	87·9	+ 0·5	3	27	198·4	193·5	+ 4·9	1
14	122·6	127·9	− 5·3	1	28	222·8	221·3	+ 1·5	2

List B

29	241·4	231·9	+ 9·5	...	32	45·5	30·3	+15·2	...
30	217·7	213·8	+ 3·9	...	33	115·6	134·5	−18·9	...
31	193·1	259·8	−66·7	...	34	92·7	91·4	+ 1·3	...

A preliminary solution of (6) and further considerations concerned with the radial velocities and parallaxes of the stars (to be dealt with later) showed that several stars have possibly little more than an exiguous claim to be considered members of the cluster; such stars are shown with an asterisk in Tables II and III, List A. These stars were omitted in the definitive solution of (6), the result of which is:

Co-ordinates of the convergent point, for 1950·0 :

$$A,\ 307°{\cdot}5 \pm 0°{\cdot}87$$
$$D,\ -39°{\cdot}3 \pm 0°{\cdot}50.$$

The results obtained by Rasmuson from the proper motions of 17 stars in his list are, brought up to 1950·0 for comparison,

$$A,\ 308°{\cdot}4 \pm 3°{\cdot}0$$
$$D,\ -39°{\cdot}7 \pm 1°{\cdot}7.$$

The increase in accuracy of the present results is to be attributed to the greatly increased accuracy of the proper motions in the *General Catalogue*.

3. Of the 21 available stars in Table II the radial velocities of 19 are given in Moore's *General Catalogue of Radial Velocities* (1932). The data are shown in Table IV. The second column gives the H.D. number of each star ; the third column gives the angular distance, λ, of each star from the convergent point ; the fourth column gives the observed radial velocities, ρ, taken from the catalogue and the next column the probable errors ; it will be noticed that several radial velocities have no probable error assigned to them and that others are either variable or suspected of being variable. In the sixth column are to be found the weights assigned in the equation of condition

$$V \cos \lambda = \rho, \tag{8}$$

from which the cluster-motion V is to be determined (as the stars are of A and F types the K-term is assumed to be negligible). A least-squares solution of (8) gives

$$V = 16{\cdot}2 \pm 1{\cdot}2 \text{ km./sec.}$$

The last column of Table IV gives the residuals. The uncertainty of many of the radial velocities and, perhaps, the somewhat speculative system of weighting make this result less dependable than that obtained in a similar way for the Taurus Cluster. For this reason the value of the cluster-velocity obtained in this section will not be used in the later calculations.

4. Following Rasmuson, we determine the cluster-motion from the known data associated with Sirius (No. 7 in Table I), the assumption being, of course, that this star is, beyond a peradventure, a member of the cluster. The arguments in favour of this method, in preference to all others, are based essentially on the well-determined total proper motion and parallax of Sirius, the probable error of each being exceedingly small in comparison with the associated quantity concerned.

If μ is the total proper motion of Sirius and p the parallax, the cluster-velocity V is given by

$$V = \frac{4{\cdot}74\mu}{p \sin \lambda}.$$

With $\mu = 1''{\cdot}324$ and $p = 0''{\cdot}375$ we find that

$$V = 19{\cdot}1 \pm 0{\cdot}2 \text{ km./sec.}, \tag{9}$$

the probable error being found by means of the tabulated probable errors in proper motion and parallax (the latter from Schlesinger's *General Catalogue of Stellar Parallaxes*, 1935) and the probable error in λ deduced from the probable errors in the co-ordinates of the convergent point.

TABLE IV

Radial Velocities

List A

No.	H.D.	λ	ρ	P.E.	Weight	Residual
		°				
4	27861	108·5	−10·7	± 2·8 ; V ?	½	− 5·6
6	40183	151·2	−18·1	...	3	− 3·9
7	48915	118·7	− 7·5	...	3	+ 0·3
9	91480	153·3	−12·4	± 0·8	3	+ 2·1
10	95418	150·2	−12·1	± 0·1 ; V ?	½	+ 2·0
11	97603	140·8	−23·2	± 1·5 ; V ?	½	−10·6
12	103287	143·4	−11·1	± 0·9	3	+ 1·9
13	106591	140·2	−12	± 4	1	+ 0·4
14	112097	117·0	− 3·8	± 2·1	1	+ 3·6
15	112185	134·8	−11·9	...	2	− 0·5
16	113139	134·0	− 7·2	± 0·8	3	+ 4·1
17	116656	130·5	− 9	...	2	+ 1·5
18	116657	130·5	− 9·2	± 0·1 ?	2	+ 1·3
19	116842	130·3	− 2	± 1·4 V	½	+ 8·5
20	118022	103·7	−10·9	± 0·5	3	− 7·1
23	134083	99·3	− 7·0	± 0·4	3	− 4·4
24	139006	95·8	+ 1·0	...	2	+ 2·6
25	141003	86·2	− 1·5	± 1·9	1	− 0·4
28	222603	59·6	+12·1	± 1·6	2	+ 3·9
				List B		
29	11171	74·8	− 0·9	± 0·8	...	− 5·8
30	38393	106·9	− 9·7	± 0·3 ; V ?	...	− 4·1
31	50204	160·7	...	...	...	...
32	79439	163·5	−16·9	± 2·3 ; V ?	...	+ 1·4
33	99028	132·5	− 9·7	± 0·4 ; V ?	...	+ 3·2
34	111456	135·9	− 9·7	± 1·5	...	+ 4·0

It seemed worth while verifying the result in (9) by extending the method to the 13 stars in List A with parallaxes not less than 0″·03. Accordingly, we have the equation of condition,

$$V \sin \lambda = 4{\cdot}74 \frac{\mu}{p}. \qquad (10)$$

As the probable error, r_μ, in μ is very much less than the probable error, r_p, in the parallax, the equations (10) were weighted according to the values of r_p tabulated in Schlesinger's catalogue. The solution is :

$$V = 18{\cdot}8 \pm 0{\cdot}7 \text{ km./sec.}$$

Table V shows the relevant details for the 13 stars (including Sirius). It will be noticed that the weight of Sirius (No. 7) is about 40 per cent. of the total weight of the 13 stars.

TABLE V

Determination of V from the Parallaxes and Proper Motions

No.	μ (Unit 0″·001)	p (Unit 0″·001)	r_p	Weight	$4{\cdot}74\mu/p$	$V \sin \lambda$	Residual
			″				
6	51	37	± 0·004	6·3	6·5	9·1	−2·6
7	1324	375	2	25·0	16·7	16·5	+0·2
9	74	33	11	0·8	10·7	8·5	+2·2
10	87	42	5	4·0	9·9	9·3	+0·6
11	201	50	5	4·0	19·1	11·9	+7·2
13	106	50	6	2·9	10·0	12·0	−2·0
16	115	30	10	1·0	18·2	13·5	+4·7
17	127	40	5	4·0	15·0	14·3	+0·7
18	123	40	5	4·0	14·6	14·3	+0·3
19	120	37	7	2·0	15·4	14·3	+1·1
23	255	63	8	1·6	19·2	18·6	+0·6
24	154	53	10	1·0	13·8	18·7	−4·9
25	86	34	± 0·007	2·0	12·0	18·8	−6·8

5. Taking the cluster-motion, V, to be 19·1 km./sec. we calculate the value of the parallax, p, by mean of (10) for the individual stars. These results are shown, for all the 34 stars of Lists A and B, in the fourth column of Table VI, the second and third columns containing the values of the total proper motion, μ, and the values of the angular distance, λ, from the convergent point. The values of the trigonometrical and spectroscopic parallaxes, taken from Schlesinger's catalogue are given in the fifth and sixth columns. The absolute magnitudes, M, derived from the formula

$$M = m + 5 + 5 \log p$$

are in the last column, the values of p used in this formula being the calculated values in the fourth column, except that an additional decimal place was retained.

Omitting the doubtful stars Nos. 1, 2, 5, 8, 21, 22 and 26, and Sirius—since, for this star, the calculated parallax is identical with the observed parallax as a result of the method adopted for determining V—and the stars Nos. 14 and 15 for which there are no determinations of trigonometrical parallax, we find from the data of Table VI that for the remaining 18 stars in List A (Nos. 1–28) the sum of the residuals, p (cal.) $- p$ (trig.), is 0″·183 from which the probable error of a residual is ± 0″·008, which is close in value to the average probable error of a determination of trigonometrical parallax.

TABLE VI

Parallaxes and Absolute Magnitudes of the Cluster Stars

No.	μ (Unit 0″·001)	λ	Parallax (Unit 0″·001)			m	Type	Absolute Magnitude, M
			Cal.	Trig.	Spectr.			
		°						
1	320	77·5	81	20	21	5·67	Fo	5·22
2	61	54·8	18	...	...	6·05	Ao	2·39
3	75	79·0	19	11	...	5·84	A2	2·23
4	75	108·5	20	9	20	5·23	A2	1·69
5	122	115·2	33	50	60	2·92	A3	0·54
6	51	151·2	26	37	68	2·07	A*op*	−0·83
7	1324	118·7	375	375	400	−1·58	Ao	1·29
8	104	74·3	27	47	...	4·18	A5	1·32
9	74	153·3	41	33	32	5·16	Fo	3·22
10	87	150·2	43	42	48	2·44	Ao	0·63
11	201	140·8	79	50	66	2·58	A3	2·07
12	94	143·4	39	17	53	2·54	Ao	0·51
13	106	140·2	41	50	50	3·44	A2	1·51
14	59	117·0	16	...	15	6·34	A3	2·41
15	114	134·8	40	...	67	1·68	A*op*	−0·32
16	115	134·0	40	30	27	4·89	Fo	2·88
17	127	130·5	41	40	43	2·40	A2*p*	0·48
18	123	130·5	40	40	43	3·96	A2	1·98
19	120	130·3	39	37	38	4·02	A5	1·98
20	50	103·7	13	18	20	4·93	A2*p*	0·47
21	58	97·1	14	8	26	3·86	A2	−0·33
22	172	98·1	43	147	140	4·64	G5	2·81
23	255	99·3	64	63	45	5·03	Fo	4·06
24	154	95·8	38	53	55	2·31	Ao	0·23
25	86	86·2	21	34	32	3·74	A2	0·39
26	54	81·1	14	...	11	6·33	F2	2·00
27	47	81·6	12	8	9	6·05	Ao	1·41
28	199	59·6	57	24	28	4·61	A5	3·40
29	175	74·8	45	...	35	4·77	Fo	3·04
30	469	106·9	122	122	120	3·80	F8	4·22
31	31	160·7	23	...	...	6·23	Ao	3·07
32	78	163·5	68	...	22	4·89	A5	4·06
33	187	132·5	63	...	49	4·03	F5	3·02
34	107	135·9	38	39	27	5·87	F5	3·77

Omitting the 7 doubtful stars, we find the average absolute magnitudes of the several spectral types, with the average residual in each case, as follows (List A only).

TABLE VII

Absolute Magnitudes and Spectral Types

	A*op*	Ao	A2*p*	A2	A3	A5	Fo
No. of stars	2	5	2	5	2	2	3
Average M	−0·57	0·81	0·47	1·56	2·24	2·69	3·39
Average residual	0·26	0·43	0·05	0·49	0·17	0·71	0·45

6. We have now three criteria for deciding whether any individual star of Table I should, or should not, be regarded as a member of the cluster. The first criterion is summarised in Table III, fourth column, which gives the differences between the observed and the calculated position-angles of the total proper motions; judged in this way, Nos. 1, 2 and 21 must be regarded as extremely doubtful members of the cluster, the remaining four of the stars with asterisks being not entirely ruled out. The second criterion is the relation between the calculated and the trigonometrical (or spectroscopic) parallax, as shown in columns 4 and 5 (or 6) of Table VI; a comparison of these columns shows that Nos. 1 and 22 must be ruled out at once as possible members of the cluster. The third criterion which is, however, not completely trustworthy, concerns the absolute magnitudes in the last column of Table VI together with the information in Table VII; it would seem that Nos. 1, 2, 5, 21 and 22 must be definitely ruled out, Nos. 8 and 26 being doubtful. The application of the three criteria evidently results in definitely discarding Nos. 1, 2, 5, 21 and 22 and in retaining Nos. 8 and 26 as possible members of the cluster.

In Table I we indicate the five discarded stars of List A by an asterisk and the two doubtful members by a double asterisk.

Of the stars in List B, Nos. 30 and 34 are members of the cluster, No. 31 is definitely not a cluster-star and the remaining three are doubtful.

7. The galactic co-ordinates (G, g) of the convergent point, for 1900·0, are

$$G = 330^\circ\cdot0, \qquad g = -37^\circ\cdot2.$$

Removing the effect of the parallactic motion * we find that, relative to the centroid of the stars near the Sun, the cluster velocity is 29·5 km./sec. towards the point $(359^\circ\cdot5, -6^\circ\cdot5)$ in galactic co-ordinates.

8. Of the 34 stars in Lists A and B, 23 would seem to be undisputed members of the cluster. With the calculated values of the parallax given in Table VI, I have derived the heliocentric galactic co-ordinates (x, y, z) in parsecs; these are given in Table VIII for the cluster-stars. Ten of these stars—Nos. 9, 10, 12, 13, 15, 16, 17, 18, 19 and 34 (of which Nos. 17 and 18 are the components of the binary ζ Ursæ Majoris)—form by themselves a comparatively compact cluster, the centre of which we take to be $(-1\cdot0, +11\cdot9, +21\cdot3)$; nine stars are within a distance of 5 parsecs of this centre. With this point as origin, the co-ordinates (ξ, η, ζ) were then found. Plots of the positions of the remaining 13 stars on the planes $x=0$, $y=0$ and $z=0$ seemed to show that the stars were roughly distributed along an axis defined by $G=0$, $g=60^\circ$. Calling this axis the Z axis and taking the X axis in the direction $(0, -30^\circ)$, with the Y axis in the direction $(90^\circ, 0)$, we find the co-ordinates (X, Y, Z) of each star from the formulæ

$$X = \xi \cos 30^\circ - \zeta \sin 30^\circ; \qquad Y = \eta; \qquad Z = \xi \sin 30^\circ + \zeta \cos 30^\circ;$$

these co-ordinates are given in Table VIII.

* We use the results obtained by the author and Mr. H. E. Green, *M.N.*, 96, 476 (Table IV), 1936, for all spectral types.

TABLE VIII

No.	x	y	z	X	Y	Z
3	−21·8	+ 9·0	−47·0	+16·1	− 2·9	−69·9
4	−41·5	+10·7	−27·8	−10·6	− 1·2	−62·7
6	−26·3	+26·3	+ 7·7	−15·1	+14·4	−24·4
7	− 2·6	− 0·7	− 0·3	+ 9·4	−12·6	−19·5
9	− 7·1	+13·0	+19·3	− 4·3	+ 1·1	− 4·7
10	− 5·5	+11·7	+19·0	− 2·8	− 0·2	− 4·2
11	− 4·6	− 1·1	+11·7	+ 1·7	−13·0	−10·1
12	− 3·4	+11·5	+22·5	− 2·7	− 0·4	− 0·2
13	− 1·7	+12·0	+21·0	− 0·5	+ 0·1	− 0·6
14	+ 2·3	−15·8	+58·9	−15·9	−27·7	+34·2
15	+ 0·6	+12·0	+22·0	+ 1·1	+ 0·1	+ 1·4
16	+ 1·0	+12·2	+22·0	+ 1·4	+ 0·3	+ 1·6
17	+ 2·3	+11·3	+21·3	+ 2·9	− 0·6	+ 1·6
18	+ 2·4	+11·6	+21·9	+ 2·6	− 0·3	+ 2·2
19	+ 2·5	+12·0	+22·5	+ 2·4	+ 0·1	+ 2·7
20	+16·7	−31·0	+69·7	− 8·9	−42·9	+50·7
23	+ 8·2	+ 0·5	+13·2	+12·0	−11·4	− 2·4
24	+15·6	+ 2·4	+20·6	+14·7	− 9·5	+ 7·8
25	+32·0	− 3·8	+33·8	+22·4	−15·7	+26·3
27	+46·0	+69·8	−13·8	+58·2	+57·9	− 6·9
28	+ 4·8	+ 8·3	−14·7	+23·0	− 3·6	−28·3
30	− 7·4	− 1·9	− 3·2	+ 6·7	−13·8	−24·4
34	+ 0·4	+14·1	+22·2	+ 0·8	+ 2·2	+ 1·5

Calling the direction of the Z axis the major axis of the distribution, we see from section 7 that the motion of the cluster is nearly at right angles to the major axis—a result in qualitative agreement with Rasmuson's conclusions, but one which should not be unduly emphasised.

I wish again to record my indebtedness to Mr. T. R. Tannahill for help in checking the calculations.

Summary

Of 34 stars suspected of being members of the Ursa Major cluster, it is concluded that 23 are cluster-stars.

The position of the convergent point is found to be :

R.A., 307°·5, Dec. −39°·3.

The parallaxes and absolute magnitudes of the cluster-stars are calculated.

The cluster-motion, referred to the mean of the stars near the Sun, is found to be 29·5 km./sec., approximately in the galactic equator.

Nearly half the stars lie within a sphere of radius 5 parsecs, and the remainder are roughly distributed along an axis towards the point (0°, 60°) in galactic co-ordinates.

University Observatory,
Glasgow, W. 2 :
1939 *February* 11.

APSIDAL MOTION IN BINARY STARS

T. E. Sterne

1. *Introduction.*—The observed ellipticities and apsidal motions of eclipsing binaries must logically depend upon the distributions of density within the components. It is important to predict as well as possible, for given distributions of density, what the expected ellipticities and rates of apsidal motion should be, in order to secure independent verification or disproof of particular distributions yielded by theories of the internal constitution. In this paper the theory of the apsidal motions * in binary stars (assumed to be constructed of compressible fluids †) will be investigated, in the limiting case where the orbital period is long compared with the free harmonic periods of the component stars. In § 2, the disturbing potential will be given ; in § 3, a general treatment will be applied to find the equilibrium tides and the disturbances, in the gravitational fields of the components, caused by those tides ; in § 4 the secular perturbations, of the longitude of periastron, caused by the disturbances in the gravitational fields, will be found from the disturbing function by conventional methods. In § 5 the results, although different from Russell's, will be shown to be consistent with them ; in § 6 they will be compared with Cowling's. The order of accuracy aimed at will be such that terms of an order up to but not including the tenth power in (a_i/A) will be retained in the analysis. The orbital eccentricity, e, will enter the analysis exactly through functions represented (*a*) by convergent power series, and alternatively (*b*) by closed expressions. The mean radii are a_1 and a_2, and the semi-major axis of the relative orbit is A.

2. *The Disturbing Potential.*—Let the components of the binary star be denoted by 1 and 2 ; let the masses be m_1 and m_2. Then the gravitational potential due to star 2 is m_2G/d at any point whose distance from the centre of mass of star 2 is d. G is the constant of gravitation. The expression is exact when star 2 is spherically constructed ; and it can be shown ‡ to be correct to terms of the order of $(m_1/m_2)\ (a_2/R)^5$ times itself at points near

* The rotational and tidal distortions will be found incidentally.

† The librations of solid stars have been considered by Walter (*Veröff. der Universitäts-Sternwarte*, Königsberg, 2, 1931 ; 3, 1933) ; the problem for fluid stars is essentially different, since no "librations" are present except those periodic in the orbital period and its submultiples. The present treatment is essentially along the lines laid down by Russell (*M.N.*, **88**, 641, 1928) except that here (*a*) the variation of the tidal deformation with distance between centres will be taken into account, and (*b*) a higher order of accuracy will be aimed at. The present treatment differs greatly from the treatment given by Cowling (*M.N.*, **98**, 734, 1938), with whose results, however, to Cowling's order of accuracy, it agrees.

‡ See equation (9) in § 4 ; or Chandrasekhar, *M.N.*, **93**, 449, 1933.

star 1, when the tidal distortion of star 2 by star 1 is taken into account. The rotational distortion of star 2, if it has a uniform angular velocity equal to the mean orbital angular velocity, introduces an error in m_2G/d of the order of $(1+m_1/m_2)\ (a_2/R)^5$ times itself at points near 1. Here R is the distance between the centres of mass. Thus to a very high order of accuracy, the simple expression m_2G/d is adequate for finding the distortion of star 1 by star 2, and the consequent disturbance, of the external gravitational potential of star 1, caused by that distortion.

In the vicinity of star 1, $(1/d)$ may be expanded in a series * of spherical harmonics about the centre of mass, Q_1, of star 1. Then the potential due to 2, at a point N whose distance from Q_1 is r, and such that the angle NQ_1Q_2 is θ, is

$$\frac{m_2G}{R}\sum_0^\infty\left(\frac{r}{R}\right)^n P_n(\cos\theta),$$

where $P_n(\cos\theta)$ is the Legendre coefficient of order n. The first term is a mere constant, producing no forces at all; the second causes an acceleration m_2G/R^2, of all the elements of star 1, parallel to the line of centres and towards 2; this acceleration is merely the orbital acceleration of star 1 as a whole. The remaining terms, constituting the tidal potential, are

$$V_t=\frac{m_2G}{R}\sum_2^\infty\left(\frac{r}{R}\right)^n P_n(\cos\theta). \qquad (0)$$

In an elliptical orbit, R is a function of the time. The general dynamical theory of the tides (produced on star 1 by the tidal potential (0)) would involve the development of each term of (0) in a series of purely harmonic functions of the mean anomaly, each multiplied by a power of the orbital eccentricity and a function of position. To each such purely harmonic function of the time there would correspond one or more partial tides; the partial tides would sweep around star 1 each with a constant amplitude, speed, and phase; the resultant of all the constantly rotating partial tides would furnish the total distortion. Such a treatment would be exceedingly complicated. In the limit, however, in which the orbital period is long compared with the free tesseral harmonic periods of the star, the results would be given by the equilibrium theory of the tides. In the general dynamical theory, the resultant distortion of star 1 must be periodic in the orbital period. In the equilibrium limit, that must still be true; and the phase differences are zero (since viscosity is small), causing the resultant tidal distortion to have at each instant the equilibrium value appropriate to V_t. The equilibrium limit will here be investigated; the results should be applicable in the absence of approaches to resonance so close as to render necessary the more general dynamical theory; and hence, in view of the expected shortness of the free tesseral harmonic periods, they should probably be applicable in practice.

The stars will here be assumed to rotate with uniform angular velocities

* See, for instance, Jeans, *Electricity and Magnetism*, Chapter VIII, 1925.

ω_1 and ω_2 about axes normal to the orbital plane *. A uniform rotation of star 1 introduces a disturbing potential

$$V_r = -\tfrac{1}{3}\omega_1^2 r^2 P_2(\cos\theta'),$$

where θ' is the co-latitude; plus a term, symmetrical about the centre of mass of star 1, which cannot distort † the star from a spherical configuration. The relevant term, V_r, added to V_t, produces a significant disturbing potential $V_r + V_t$.

If U is the gravitational potential of the disturbed gaseous (or liquid) star, and V_d is the complete disturbing potential, so that $\Psi = U + V_d$ is the total potential, then the equation of hydrostatic equilibrium is $dP = \rho d\Psi$ where P is the pressure and ρ the density. It follows immediately that P and ρ must be functions of Ψ alone in a star that is in hydrostatic equilibrium under the action of its own gravity and of a disturbing potential.

3. *The Deformation and the Consequent External Gravitational Potential.*— The elegant and general treatment given by Jeffreys ‡ may be adapted to this problem. Let the disturbing potential, arising from sources other than the attraction of star 1 on itself, be expanded in a series of tesseral harmonic terms of the form

$$c_{n,m} r^n P_n^m(\theta, \phi), \qquad \text{(A)}$$

where $n \geqslant 2$; r is the distance from the centre of mass of star 1; P_n^m is a tesseral surface harmonic function of θ and ϕ, polar co-ordinates taken with respect to the centre of mass of star 1; and $c_{n,m}$ is a constant. One considers the form of the surface, over which the density is ρ in the distorted star 1, to be given by the equation

$$r = r_1\Big(1 + \sum_{n,m} Y_n^m\Big), \qquad (1)$$

where r_1 is a parameter (the mean radius) characterizing the surface in

* Although it is uncertain that actual components so rotate, there is no general agreement as to how they do rotate, so that the assumptions are probably justifiable, at present, on grounds of simplicity. Some assumption has to be made about the variation of ω with depth, and that of uniform rotation is the simplest. The additional complications that would follow from the consideration of inclined axes of rotation would at present, moreover, not be justifiable. See Jeans, *Astronomy and Cosmogony*, Chapter X, 1929; also Rosseland, *Astrophysica Norvegica*, **2**, 173, 1936 and **2**, 249, 1937. Rosseland's solutions are sufficient, not necessary.

† The term can, and in general probably does, produce a swelling or contraction of the star, compared with the equilibrium configuration of an undisturbed star having the same mass and luminosity. Provided that such effects are taken into account in the "model" to which equation (5) of the next section is applied, it is legitimate to ignore such effects here. For a treatment of them, in relation to some particular stellar models, the reader is referred to Chandrasekhar, *M.N.*, **93**, 390, 1933. The reason for its being permissible to regard the term in the present fashion is that our treatment gives information about the expected apsidal motion corresponding to a given distribution of density in the actual, distorted, star; not in some ideal undistorted star.

‡ Jeffreys, *The Earth*, Chapter XIII, 1924. The general procedure appears to have been devised by Laplace, Clairaut, and Radau.

question, and the Y_n^m's are tesseral surface harmonic functions of θ and ϕ, and are also functions of r_1. Let the distorted outer boundary of star 1 correspond to $r_1 = a$. Then if one ignores small quantities of the order of the squares and cross products of the Y's there holds for Y_n^m the equation *

$$-\frac{Y_n^m}{r_1}\int_0^{r_1}\rho' a'^2 da' + \frac{1}{(2n+1)r_1^{n+1}}\int_0^{r_1}\rho'\frac{\partial}{\partial a'}(a'^{n+3}Y_n^m)da' + \frac{r_1^n}{2n+1}\int_{r_1}^{a}\rho'\frac{\partial}{\partial a'}\left(\frac{Y_n^m}{a'^{n-2}}\right)da' = -\frac{1}{4\pi G}c_{n,m}r_1^n P_n^m(\theta,\phi). \quad (2)$$

Here the quantity a' is the value of r_1 for which $\rho = \rho'$. Multiply both sides of equation (2) by r_1^{n+1}, differentiate with respect to r_1, and divide by r_1^{2n}. One obtains

$$-\left[\frac{nY_n^m}{r_1^{n+1}} + r_1^{-n}\frac{\partial Y_n^m}{\partial r_1}\right]\int_0^{r_1}\rho' a'^2 da' + \int_{r_1}^{a}\rho'\frac{\partial}{\partial a'}\left(\frac{Y_n^m}{a'^{n-2}}\right)da' = -\frac{1}{4\pi G}(2n+1)c_{n,m}P_n^m(\theta,\phi). \quad (3)$$

Differentiate with respect to r_1:

$$\frac{\partial^2 Y_n^m}{\partial r_1^2} - \frac{n(n+1)}{r_1^2}Y_n^m + \frac{6\rho}{S(r_1)}\left(r_1^2\frac{\partial Y_n^m}{\partial r_1} + r_1 Y_n^m\right) = 0, \quad (4)$$

where

$$S(r_1) = 3\int_0^{r_1}\rho' a'^2 da'.$$

Now if one introduces a new variable

$$\eta_n = (r_1/Y_n^m)(\partial Y_n^m/\partial r_1),$$

a function of r_1, equation (4) reduces to

$$r\frac{d\eta_n}{dr} + \eta_n^2 - \eta_n - n(n+1) + \frac{6\rho}{\rho_m}(\eta_n + 1) = 0, \quad (5)$$

where r_1 has been replaced (there should be no danger of confusion) by r. Here ρ is the density at the distance r from the centre, and ρ_m is the mean density interior to r, in a spherical distribution of matter (not necessarily in equilibrium) in which the density is the same function of r that ρ, in the actual distorted star, is of r_1. It is shown by Jeffreys † that at $r = 0$, $\eta_n = n - 2$.

In (3), let $r_1 = a$, the mean radius of the star. Then it is found that the term (A) in the disturbing potential produces a term Y_n^m in (1) whose value at the surface is

$$Y_n^m(a) = \frac{2n+1}{n+\eta_n(a)}\frac{1}{m_1 G}c_{n,m}a^{n+1}P_n^m(\theta,\phi), \quad (6)$$

* Since Ψ must reduce to a function of r_1 alone. See Jeffreys, *ibid.*, equation (2) of § 13.4.

† Jeffreys, *ibid.*, § 13.52.

where $\eta_n(a)$ is the value of η_n at the outer surface. Then set $r_1 = a$ in (2) and make use of the appropriate expression * for the gravitational potential (due to the distortion) at external points. One then finds that the distortion of star 1 produced by the term (A) in the disturbing potential gives rise to a contribution

$$U_{n,m} = \frac{n+1-\eta_n(a)}{n+\eta_n(a)} c_{n,m} \frac{a^{2n+1}}{r^{n+1}} P_n^m(\theta, \phi), \tag{7}$$

to the gravitational potential, at external points, of star 1.

The external shape is given by

$$r = a\left(1 + \sum_{n,m} Y_n^m(a)\right), \tag{6 a}$$

where the separate terms are given by (6). The total contribution of the distortion of star 1 to the external gravitational potential of star 1 is the sum of the separate terms $U_{n,m}$ given by (7).

For any model, equation (5) may be solved, numerically or otherwise, so as to find the value $\eta_n(a)$ of η_n at the boundary $r = a$. Then equations (6) and (6 *a*) give the superficial distortion of star 1 ; and the equations (7) give the contribution of the whole "internal" distortion of star 1 to its external gravitational potential.

We have introduced tesseral harmonics explicity in order to make simpler the practical applications of these results later on. For the preceding results show that to any term, in the disturbing potential, of the form

$$cr^n P_n, \tag{B}$$

where P_n is a surface harmonic of order $n \geqslant 2$, there corresponds a contribution to the superficial distortion given by

$$Y_n(a) = c \frac{2n+1}{n+\eta_n(a)} \frac{a^{n+1}}{m_1 G} P_n, \tag{6 b}$$

and a contribution to the external gravitational potential of star 1

$$U_n = c \frac{a^{2n+1}}{r^{n+1}} \frac{n+1-\eta_n(a)}{n+\eta_n(a)} P_n. \tag{7 b}$$

Different P_n's need not be referred to the same axes ; and the Y_n and U_n always have the same character as the surface harmonic, P_n, present in the disturbing term (B) that causes them.

Although but little use will be made in this paper of the equations (6) and (6 *b*) that furnish the superficial distortion, they follow immediately from the analysis and have been given for completeness.

The results obtained in this section will now be applied to the disturbing potential $V_t + V_r$.

* Obtained, simply, by replacing each Y_n', in Jeffreys' equation (3) of § 13.12, by a linear combination of terms Y_n^m in different m's.

4. *The Secular Motion of the Apse.*—The terms of order $n=2$ in the disturbing potential V_r+V_t that acts on star 1 have been found to be

$$\frac{m_2G}{R^3}\, r^2P_2(\cos\theta)-\tfrac{1}{3}\omega_1{}^2r^2P_2(\cos\theta'). \tag{8}$$

From equation (7 *b*), these terms so distort * star 1 as to contribute to its external gravitational potential at all points the terms

$$2\frac{a_1{}^5}{r^3}k_{2,1}\left[\frac{m_2G}{R^3}P_2(\cos\theta)-\frac{\omega_1{}^2}{3}P_2(\cos\theta')\right]. \tag{9}$$

For the centre of mass of star 2, $r=R$, $\theta=0$ and $\theta'=\pi/2$; the acceleration of star 2 thus contains the terms, since $P_2(x)=\frac{1}{2}(3x^2-1)$,

$$6\frac{a_1{}^5}{R^4}k_{2,1}\left[\frac{m_2G}{R^3}+\frac{\omega_1{}^2}{6}\right],$$

directed towards the centre of mass of star 1. These last terms correspond to a potential function

$$\psi_2=a_1{}^5k_{2,1}\left[\frac{m_2G}{R^6}+\frac{\omega_1{}^2}{3R^3}\right]. \tag{9 a}$$

In equations (9) and (9 *a*), $k_{2,1}$ (denoted by k_1 in Russell's paper) is the quantity

$$\frac{3-\eta_2(a)}{4+2\eta_2(a)}$$

computed for star 1. Here, by equation (5), $\eta_2(a)$ is the value at $r=a$ of the variable η_2 that is zero at $r=0$ and that satisfies the first order differential equation

$$r\frac{d\eta_2}{dr}+\eta_2{}^2-\eta_2-6+\frac{6\rho}{\rho_m}(\eta_2+1)=0. \tag{10}$$

The terms (9 *a*) correspond to a contribution $\psi_2(1+m_2/m_1)$ to the ordinary disturbing function S of perturbation theory, when the elements of the relative orbit are under consideration. The rate of motion of the apse is known to be given by the equation, linear in S,

$$\dot{\varpi}=(1-e^2)^{1/2}\frac{1}{e}\frac{\partial S}{\partial e}(\mu A)^{-1/2}, \tag{11}$$

* As an illustration of the exceedingly great generality of the analysis described in the preceding section, equation (6 *b*) may be used to compute the distortion of a polytrope of index three under the action of the terms (8). The value $k_2=0{\cdot}0144$ for that model was computed by Russell (*ibid.*) from equation (5). Equation (6 *b*) yields a value of Δ_2, a coefficient of P_2 in Chandrasekhar's special treatment of the tidal distortion (*M.N.*, **93**, 456, 1933) of the model in question, equal to 1·0288 ; whereas Chandrasekhar's own value, reached by his special analysis, is 1·0289. Chandrasekhar's Δ_2 should equal $2k_2+1$. Likewise, equation (6 *b*) leads, with Russell's value of k_2, to a rotational oblateness $41{\cdot}810\ v$, where Chandrasekhar's special treatment leads to the value $41{\cdot}811\ v$ (*M.N.*, **93**, 399, 1933).

where e is the orbital eccentricity, A is the semi-major axis of the relative orbit, $\mu = G(m_1 + m_2)$, and S is supposed to be defined * as a function of the mean anomaly M, of e, and of A.

The quantities R^{-3} and R^{-6}, appearing in $\dot{\psi}_2$, and thus in S, must be developed in terms of M, e, and A. Although $(A/R)^j$ can be expanded in terms of Hansen's coefficients, it is perhaps simpler to consider the expansion, involving † Bessel's coefficients,

$$\frac{A}{R} = 1 + 2\sum_{p=1}^{\infty} \mathfrak{J}_p(pe) \cos pM.$$

If this expansion is raised to the j'th power, an infinite series in powers of e will be obtained, plus a set of such series each multiplied by a cosine of a multiple of M. The whole expression is to be operated on by $(1/e)(\partial/\partial e)$, and it is seen that the periodic terms will contribute merely periodic terms to the velocity $\dot{\varpi}$ of the apse. The secular motion of the apse arises from the series in e^2 (odd powers are lacking) alone, and the lowest terms of importance are those in e^2 itself, which contribute pure constants to the secular motion. The series in e^2 can be easily found, for

$$(A/R)^j = (1 - e\cos E)^{-j}$$

$$= \sum_0^{\infty} e^g \frac{(j+g-1)!}{(j-1)!\,g!} \cos {}^gE,$$

where E is the eccentric anomaly. When g is even, the expression for $\cos {}^gE$ in terms of cosines of multiples of E is a constant term $g!/2^g[(g/2)!]^2$ plus cosine terms in even multiples of E; when g is odd, the expression is $g! \cos E/2^{g-1}[(g-1)/2]![(g+1)/2]!$ plus cosine terms in odd multiples of E. Since the development of $\cos mE$ in cosines of multiples of M has as its only constant term 1, $-\frac{1}{2}e$, or 0 according to whether $m = 0$, $m = 1$, or $m > 1$, it follows that the coefficient of e^k in the non-periodic part of $(A/R)^j$ is

$$b(j, k) = \frac{(j+k-2)!}{(j-2)!\,2^k(k/2)!^2}$$

when k, which must be even, is larger than 0; the coefficient of e^0 is unity. The non-periodic part of $(A/R)^j$ can thus be written down at once as a series, in e^2, convergent for all values of e less than unity. One finds, thus,

$$\left(\frac{A}{R}\right)^3 = 1 + \frac{3}{2}e^2 + \frac{15}{8}e^4 + \frac{35}{16}e^6 + \frac{315}{128}e^8 + \ \ldots\ + \text{periodic terms},$$

$$\left(\frac{A}{R}\right)^6 = 1 + \frac{15}{2}e^2 + \frac{105}{4}e^4 + \frac{525}{8}e^6 + \frac{17325}{128}e^8 + \ \ldots\ + \text{periodic terms}.$$

* The equation for $\dot{\varpi}$ given above may be obtained readily by considering variations of the canonical constants of an orbit in a plane. Generally, in such an orbit, S is a function of ϵ (the mean longitude at the epoch), of ϖ (the longitude of periastron), of A, and of e, as well as of the time. For present purposes, since only $(\partial S/\partial e)$ is involved, one may regard S as a function simply of M, e, and A.

† See Plummer's *Dynamical Astronomy*, Chapter IV.

It follows from these expansions, and from equations (9 *a*) and (11), that distortions of order 2 of star 1 contribute to the secular motion $\dot{\varpi}_s$, of the apse terms

$$k_{2,1}a_1^5\frac{1}{m_1}\frac{(m_1+m_2)^{1/2}}{(GA)^{1/2}}\left[15\frac{m_2G}{A^6}f_2(e)+\frac{\omega_1^2}{A^3}g_2(e)\right], \tag{12}$$

where $f_2(e)$ and $g_2(e)$ are series, convergent for any value of e less than unity,

$$f_2(e)=1+\frac{13}{2}e^2+\frac{181}{8}e^4+\frac{465}{8}e^6+\ \ldots \tag{12 a}$$

$$g_2(e)=1+2e^2+3e^4+4e^6+\ \ldots \tag{12 b}$$

Making use of the relation $\nu^2A^3=\mu$, where ν is the mean orbital motion, one finds for the contribution to $\dot{\varpi}_s/\nu$

$$k_{2,1}\frac{a_1^5}{A^5}\left[15\frac{m_2}{m_1}f_2(e)+\frac{\omega_1^2A^3}{m_1G}g_2(e)\right]. \tag{13}$$

If one now considers the perturbations due to the action on star 1 of the distortion (caused by the attraction of star 1 and by the rotation of star 2) of star 2, one sees that the second order harmonic distortions give rise to a secular apsidal motion (a precession) $\dot{\varpi}_2$ such that

$$\begin{aligned}\frac{\dot{\varpi}_2}{\nu}=&k_{2,1}\frac{a_1^5}{A^5}\left[15\frac{m_2}{m_1}f_2(e)+\frac{\omega_1^2A^3}{m_1G}g_2(e)\right]\\&+k_{2,2}\frac{a_2^5}{A^5}\left[15\frac{m_1}{m_2}f_2(e)+\frac{\omega_2^2A^3}{m_2G}g_2(e)\right],\end{aligned} \tag{14}$$

where, as usual, the subscripts 1 and 2 refer to stars 1 and 2 respectively.

Should one wish to set the ω's equal to ν, one obtains the less general result

$$\begin{aligned}\frac{\dot{\varpi}_2}{\nu}=&k_{2,1}\frac{a_1^5}{A^5}\left[\frac{m_2}{m_1}(15f_2(e)+g_2(e))+g_2(e)\right]\\&+k_{2,2}\frac{a_2^5}{A^5}\left[\frac{m_1}{m_2}(15f_2(e)+g_2(e))+g_2(e)\right].\end{aligned} \tag{14 a}$$

The contributions arising from the third and fourth order harmonic distortions may be dealt with similarly. The terms of these orders in the disturbing potential are purely tidal, arising from the tidal potential (0). The disturbing term of order $n=3$ is

$$\frac{m_2G}{R^4}r^3P_3(\cos\theta)$$

and by (7 *b*)

$$\psi_3=k_{3,1}\frac{m_2G}{R^8}a_1^7,$$

where $k_{3,1}$ is the quantity

$$\frac{4-\eta_3(a)}{6+2\eta_3(a)}$$

and where $\eta_3(a)$ is obtained by solving equation (5), with n set equal to 3, and subject to the boundary condition $\eta_3 = 1$ at $r = 0$, for the value of η_3 at the boundary of star 1. The contribution to the disturbing function is $\psi_3(1 + m_2/m_1)$, and if one makes use of the expansion of $(A/R)^8$ one finds for the contribution $\dot{\omega}_3$ of the third order deformations to the secular motion of the apse

$$\frac{\dot{\omega}_3}{\nu} = 28\left[k_{3,1}\frac{a_1^7}{A^7}\frac{m_2}{m_1} + k_{3,2}\frac{a_2^7}{A^7}\frac{m_1}{m_2}\right]f_3(e), \tag{15}$$

where

$$f_3(e) = 1 + \tfrac{43}{4}e^2 + \tfrac{449}{8}e^4 + \tfrac{12941}{64}e^6 + \quad . \quad . \quad . \tag{15\,a}$$

The disturbing term of order $n = 4$ is

$$\frac{m_2 G}{R^5} r^4 P_4(\cos\theta)$$

and by (7 *b*)

$$\psi_4 = k_{4,1}\frac{m_2 G}{R^{10}} a_1^9,$$

where $k_{4,1}$ is the quantity

$$\frac{5 - \eta_4(a)}{8 + 2\eta_4(a)},$$

and where $\eta_4(a)$ is given by equation (5), with n set equal to 4, subject to the boundary condition that $\eta_4 = 2$ at $r = 0$. From the expansion of $(A/R)^{10}$ one finds for the contribution $\dot{\omega}_4$ of the fourth order deformations to the secular motion of the apse

$$\frac{\dot{\omega}_4}{\nu} = 45\left[k_{4,1}\frac{a_1^9}{A^9}\frac{m_2}{m_1} + k_{4,2}\frac{a_2^9}{A^9}\frac{m_1}{m_2}\right]f_4(e), \tag{16}$$

where

$$f_4(e) = 1 + 16e^2 + \tfrac{467}{4}e^4 + \tfrac{8975}{16}e^6 + \quad . \quad . \quad . \tag{16\,a}$$

Still higher orders in (0) could be taken into account; but one would then have to take into account also the effect upon the apsidal motion of the distortion of each star produced by the distortion of the other star; these effects should introduce terms in $\dot{\omega}_s/\nu$ of the order of $(a_i/A)^{10}$. The results given here, for $\dot{\omega}_s/\nu$ in the equilibrium limit, should be correct up to but not including terms of the tenth order in (a_i/A). To the present order of accuracy, since $\dot{\omega}_s/\nu = P/P'$,

$$\frac{P}{P'} = \frac{\dot{\omega}_2}{\nu} + \frac{\dot{\omega}_3}{\nu} + \frac{\dot{\omega}_4}{\nu}, \tag{17}$$

where P and P' are the orbital and apsidal periods, respectively, and where the $\dot{\omega}/\nu$'s are given by equations (14), (15), and (16) if no assumptions are made about the ω's, or by (14 *a*), (15), and (16) if the ω's are set equal to ν. For well separated components, the term $\dot{\omega}_2/\nu$ alone, given by (14) or (14 *a*), should yield a sufficiently accurate value of P/P'. The present theory always predicts a precession.

Values of k_2 have been tabulated for a series of models by Russell *. For homogeneous stars k_2 has the value 3/4 ; for polytropes of index three, the value 0·0144 ; for completely concentrated stars all the k's are zero. For homogeneous stars, k_3 and k_4 have the values 3/8 and 1/4, respectively. All the k's can readily be found for any model by numerical integration of equation (5).

In the case of certain polytropic models, however, the integrations need not be carried out, for the k's have in effect been computed, under a different guise, by Chandrasekhar †. If equations (6 *a*) and (6 *b*) are applied to the tidal potential (0), one finds directly, for the equilibrium shape of star 1 produced by the tidal ‡ action,

$$\frac{r-a_1}{a_1}=\frac{m_2}{m_1}\left[\frac{5}{2+\eta_2(a)}z^3P_2+\frac{7}{3+\eta_3(a)}z^4P_3+\frac{9}{4+\eta_4(a)}z^5P_4\right],$$

where P_j denotes $P_j(\cos\theta)$ and where $z=(a_1/R)$. In the present notation, Chandrasekhar's equation (34) is

$$\frac{r-a_1}{a_1}=\frac{m_2}{m_1}\sum_2^4\Delta_j z^{j+1}P_j.$$

The corresponding coefficients must be equal ; from this it follows that $k_j=\frac{1}{2}(\Delta_j-1)$. Since Chandrasekhar lists the Δ_j's for the polytropes of indices 1, 3/2, 2, 3 and 4, one has at once the values

Polytropic index	1	1·5	2	3	4
k_2	0·25992	0·1446	0·0741	0·0144	0·00134
k_3	0·1064	0·0540	0·0244	0·00368	0·00024
k_4	0·0602	0·0281	0·0116	0·00140	0·00007

The value of k_2 obtained in this way, for the polytrope of index three, agrees exactly with the value computed by Russell directly from equation (5).

The series expansions (12 *a*), (12 *b*), (15 *a*) and (16 *a*) do not converge very rapidly for values of e larger than a few tenths. Closed expressions may be employed instead. For $g_2(e)=(1/3)F_3(e)$; $f_2(e)=(1/15)F_6(e)$; $f_3(e)=(1/28)F_8(e)$; and $f_4(e)=(1/45)F_{10}(e)$ where

$$F_j(e)=\frac{j}{2\pi e}(1-e^2)^{1/2}\int_0^{2\pi}\frac{\cos E-e}{(1-e\cos E)^{j+1}}dE.$$

The integral may be evaluated by the substitution

$$\cos\theta=(\cos E-e)/(1-e\cos E)\,;$$

---

* Russell, *ibid.*

† Chandrasekhar, *M.N.*, **93**, 456, 1933.

‡ The rotational distortion

$$\frac{r-a_1}{a_1}=-\frac{5}{2+\eta_2(a)}\frac{\omega_1^2a_1^3}{3m_1G}P_2(\cos\theta'),$$

given by (6 *b*), is not needed here but is given for completeness.

and it is then found, exactly for all values of e less than unity, that

$$\left.\begin{aligned} g_2(e) &= (1-e^2)^{-2}, \\ f_2(e) &= (1-e^2)^{-5}(1+\tfrac{3}{2}e^2+\tfrac{1}{8}e^4), \\ f_3(e) &= (1-e^2)^{-7}(1+\tfrac{15}{4}e^2+\tfrac{15}{8}e^4+\tfrac{5}{64}e^6), \\ f_4(e) &= (1-e^2)^{-9}(1+7e^2+\tfrac{35}{4}e^4+\tfrac{35}{16}e^6+\tfrac{7}{128}e^8). \end{aligned}\right\} \qquad (18)$$

The functions $g_2(e)$ and the $f(e)$'s have the values unity when $e=0$; are rapidly increasing functions of e; and approach infinity, thereby causing the secular precession of the apse to approach infinity, as e approaches unity.

5. *Comparison with Russell's Results.*—Russell set the ω's equal to ν, considered the limit of vanishing e, ignored tidal deformations of higher order than $n=2$, and considered the amplitude of the tidal deformation to be fixed at the value corresponding to a distance between centres equal to the semi-major axis A. His equation for P/P' should therefore be comparable to, although not identical with, equation (14 a) when the f_2 and g_2 in that equation are set equal to unity, since (14 a) then carries the analysis to Russell's order of accuracy. Russell's equation then in fact differs from (14 a) only in containing the numerical factor 7 where the factor 16 appears in (14 a). The amount of this discrepancy is exactly what one should expect. A comparison of equations (14) and (14 a) shows that 16 is the sum of unity, arising from the rotation, and 15 arising from the tides; Russell's 7 is likewise the sum of unity, from the rotation, and 6 from the tidal deformation. Russell's treatment and the present treatment yield, as they should, the same rotational contributions to the apsidal motion; the present treatment yields a tidal precession five-halves as large as Russell's. This it should do; for ignoring the variation of the tidal distortion with time, during the orbital motion, amounts to setting R, in equation (9), equal to A. When the operator $(1/e)(\partial/\partial e)$ is later applied to $2(A/R)^3$ instead of to $(A/R)^6$, a tidal precession must be obtained exactly two-fifths as large in the former case as in the latter, since the coefficient of e^2 in the former expansion is 3, and in the latter 15/2. The present results * are therefore consistent with Russell's.

For any given model, the present treatment leads to an expected rate of apsidal precession that is, roughly, about twice as rapid as Russell's would predict. Hence, when applied to the same observational data, the present treatment implies a considerably higher degree of central condensation in the components than does Russell's.

6. *Comparison with Cowling's Results.*—A treatment very different from the present one has been given recently by Cowling †. Cowling's treatment does not take into account deformations of higher order than those corresponding to $n=2$, and so his equations (40) and (41) should be, respectively,

* That the consideration of varying tides should lead to such different results from the assumption of constant tides is due to the circumstance that terms in e^2 in the disturbing function (although they approach zero as e approaches zero), since they yield mere constants after the operator $(1/e)\ (\partial/\partial e)$ is applied to them, are for small eccentricities the most important terms, in producing secular apsidal motions, in the disturbing function.

† Cowling, *ibid.*

the same as equations (14) and (14 *a*) here. Equations (14) and (14 *a*) then carry the results to Cowling's order of accuracy and are seen to be the same as his.

The present analysis leads to considerably higher degrees of central condensation than Cowling's, when applied to eclipsing binaries for which (a_i/A) is large.

7. *Summary.*—The secular motion of the apse, in the orbits of binary stars whose components are assumed to be composed of compressible fluids, is found in the limiting case where the orbital period is long compared with the free harmonic periods of the components. Account is taken of the variation of the tidal deformations with time. Terms of an order up to but not including the tenth power of (a_i/A), where the mean radii are a_1 and a_2 and the semi-major axis of the relative orbit is A, are retained in the final results. The orbital eccentricity, e, enters the analysis exactly through functions which are represented by closed expressions valid for all values of e less than unity. The motion is found to be a precession, and the results are the same as Cowling's (*M.N.*, **98**, 734, 1938) to his order of accuracy, which includes terms of the fifth order in (a_i/A). The present more complete results, which are given in a form applicable to all models, lead to still higher degrees of central condensation than Cowling's when (a_i/A) is not very small. The superficial tidal and rotational distortions are found incidentally, in forms applicable to all stellar models. For given stellar models, masses, a_i's, rotations, and A, the predicted apsidal precession increases rapidly with e and approaches infinity as e approaches unity.

Harvard College Observatory :
1938 *December* 11.

A SIMPLE WEDGE-PHOTOMETER FOR OBSERVING BRIGHT CHROMOSPHERIC ERUPTIONS WITH A SPECTRO-HELIOSCOPE

H. W. Newton

(Communicated by the Astronomer Royal)

1. The relationship recently established between bright chromospheric eruptions and sudden ionospheric disturbances has made it necessary to obtain further data concerning the intensity of the solar eruptions. Hitherto, the relative intensities of the eruptions have been estimated on a visual scale of 1 (smallest)–2–3 (greatest). Although these eye estimates have been productive of useful results, definitive observations of intensity are now required, and these involve photometric measures combined perhaps with measures of area. In this experimental stage, it is best to keep the accessory apparatus as simple as possible, so that it can be easily set up and tested on any spectrohelioscope. Speed of working rather than great accuracy is required, because the phenomena under investigation vary rapidly with time.

2. Since 1932, visual measures of the peak intensities of bright *Hα* flocculi have been made at Greenwich with

(1) a simple, direct-wedge photometer *, and later with
(2) a lamp photometer† .

The advantages of the direct-wedge are :

(*a*) Simplicity.
(*b*) The comparison ‡ between flocculus and the adjacent background is direct, and short breaks in a cloudy sky can therefore be utilized.

The disadvantages are :

(*c*) The measures give a ratio only, namely, peak intensity of flocculus to background. The instrumental intensity of this background (the centre of *Hα*) as compared with that of the adjacent continuous spectrum has to be determined.
(*d*) Unsuitability for observing prominences at the Sun's limbs, where limb darkening is operating in the adjacent background.

* *M.N.*, **93**, 172, 1933.

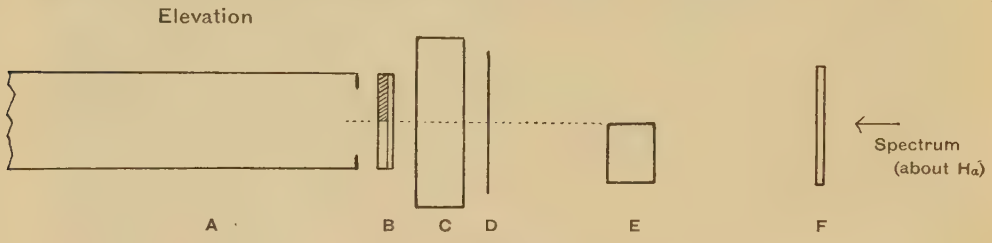
† *M.N.*, **96**, 5, 1935. Photometric measures from spectroheliograms taken at Cambridge of two bright eruptions are given by Dobbie, Moss and Thackeray in *M.N.*, **98**, 606, 1938.

‡ The comparison is usually made in the centre of *Hα*, 6562·8 A., since the average bright flocculus gives either little or no Doppler displacement.

The disadvantages (*c*) and (*d*) were overcome by the use of a lamp photometer, in which the lamp spectrum is used as an intermediary in the following intensity comparison : (1) continuous spectrum of disc outside the wings of $H\alpha$: (2) peak intensity of flocculus: (3) continuous spectrum, as before, to check stability of sky and lamp. The region of the disc observed for continuous spectrum may be either adjacent to the flocculus or at the centre of the disc as in the case of prominences. With the lamp photometer it was also possible to observe visually the contour of $H\alpha$, and partial contours have also been made of prominences and absorption markings (*M.N.*, **96**, 8, 1935).

Advantage (*b*) of the direct-wedge method is lost, however ; one complete comparison (disc—flocculus—disc) takes a few minutes, and a steady sky is essential.

3. The addition of a secondary line-shifter for use with the direct-wedge photometer (a suggestion due to Mr. C. R. Davidson) will enable a comparison of the peak intensity of a flocculus to be made directly with the



Arrangement of Apparatus.

adjacent *continuous* spectrum. With this simple modification, the direct-wedge photometer seems to be very suitable for visual measures of the intensity of bright eruptions. The arrangement of the optical system is as follows :—

A Eyepiece of spectrohelioscope.
B Neutral graduated wedge.
C Lower rotating prism (square in cross-section).
D Second slit of spectrohelioscope.
E Subsidiary line-shifter as proposed.
F Primary line-shifter.

A subsidiary line-shifter *E*, capable of giving a considerable displacement, say of 15 A. when turned about a vertical axis from its zero position to about 45°, is inserted in the optical train of the spectrohelioscope between the original line-shifter *F* and the second slit *D*. *E* is made one-half the height of *F*. Suppose that both *E* and *F* are set normal to the incoming beam, and that the grating is adjusted so that $H\alpha$ light from the Sun's image passes the second slit *D*. The rotating prisms, of which *C* is the lower component, will give a steady image of a part of the Sun's disc in λ 6563 as viewed in the eyepiece *A*. The image of the hydrogen flocculus, whose intensity it is intended to measure, is brought just above the upper half of

the field viewed through the wedge *B*. The subsidiary line-shifter *E* is then turned clock-wise through a predetermined angle, so that the *Hα* line is displaced by 15 A. from the lower half of slit *D*. This half of the slit will now only pass light of λ 6578 *, while that of the top half is still passing light of λ 6563 in which the flocculus is visible. The graduated wedge *B* is then adjusted until equality in brightness is obtained between the flocculus and the continuous spectrum. The wedge readings when corrected for the effective absorption of the subsidiary line-shifter *E* give the ratio :

peak intensity of flocculus/intensity of continuous spectrum.

The effective absorption of *E* is readily measured by comparing two adjacent areas of continuous spectrum 15 A. apart not far from *Hα*, the one passing through the wedge *B*, the other through *E*. The measure of effective absorption will include any difference in the sensitivity of the eye to light of *Hα* and of *Hα* ± 15 A.

The neutral wedge used at Greenwich was supplied by Messrs. Ilford, Ltd., and has a range of density from 0 to 1·5. The wedge (No. 1109) is 15 cm. long and 1 cm. wide, and with a piece of clear glass as a compensator is bound together with a common cover glass, making the overall dimensions 15 cm. × 2 cm. The wedge is mounted in a frame held in position by a clamp between the eyepiece and the rotating prisms. If a single continuous wedge is used, as at Greenwich, it is desirable to limit the width of the field of view by a slit of 2 or 3 mm. in width. If a step wedge is used, this precaution is not necessary.

The subsidiary line-shifter is a piece of crown glass 10 mm. in thickness. This need not be mounted on a graduated turntable ; when once the correct position has been determined for a resulting displacement of 15 A., a stop may be fixed for bringing the line-shifter up to register. For the estimation of areas, provision is made for a graticule to be inserted between *B* and *C*.

In conclusion it may be added that the observations of intensity obtained in this manner will require corrections on account of finite resolving power of the instrument and for scattered light. These corrections applicable to spectrohelioscopic observations are discussed by Woolley in *M.N.*, **96**, 10, 1935, and **97**, 531, 1937. Each observer must eventually determine the corrections necessary for his own instrument, but uncorrected observations of bright eruptions will give the order of magnitude and the time-intensity curve for any particular eruption. Since by means of the subsidiary line-shifter an intensity square, representing the adjacent continuous spectrum, is easily made accessible in the field of the eyepiece, unusually bright eruptions approaching the intensity of the continuous spectrum can be detected at a glance.

* There is a convenient region here free of spectral lines. On the short-wave side of *Hα* a displacement of 12 A. is convenient. According to Thackeray's determination of the contour of *Hα* (*M.N.*, **95**, 295, 1935) the wings extend beyond 10 A. from the centre. The value given is 0·97, that of the continuous spectrum at distances of +10 A. and −15 A. respectively from the centre of the line. *Cf.* Minnaert, *Zeits. f. Physik*, 45, 610, 1927.

Summary

A description is given of a simple wedge-photometer intended especially for the rapid measurement of intensities of bright chromospheric eruptions observed with a spectrohelioscope. The wedge comparison is made directly in the field of view between the bright area as seen in $H\alpha$ and the continuous spectrum, 15 A. away from the centre of $H\alpha$, coming from a contiguous region of the disc. This juxtaposition in the field of adjacent areas of the Sun seen respectively in $H\alpha$ and in $H\alpha \pm 15$ A. is effected by introducing into the optical train a thick secondary line-shifter half the height of the primary one.

Royal Observatory, Greenwich :
1939 *February* 27.

SPACE REDDENING DEDUCED FROM THE MOVING CLUSTER IN PERSEUS

E. G. Martin

(Communicated by the Astronomer Royal)

In *M.N.*, **71**, 43, 1910, Sir Arthur Eddington gave particulars of 27 B-type stars in the region 2^h to 6^h R.A. and $+36°$ to $+70°$ Dec. Sixteen of these stars had proper motions of about $4''$ per century with common direction of motion, position angle about 135°, whilst ten had proper motions of $1''{\cdot}5$ per century. The remaining star had a larger motion and had moved beyond the limits of the diagram showing the two groups. The selection of stars was made by taking stars of B-type from *H.A.* **50** and proper motions from Boss's *P.G.C.*

It is now possible with later information to re-examine these stars. Improved spectral types, parallaxes, and colour determinations are now available. The region has been slightly extended south to $+30°$ Dec., and a few late O-type stars included.

In the table which follows, the 16 fast-moving stars are given, together with 4 additional stars of similar proper motion. The mean parallax and proper motion of the group are given at the foot of the appropriate column.

Stebbins and Huffer*, using a photo-electric cell in conjunction with filters at the Washburn Observatory, have determined the colour index of many B-type stars. Column 9 gives the colour excess, C.E., obtained by differencing the colour index from the mean colour index of normal stars of that spectral type. The normal stars excluded stars nearer than 15° from the galactic equator and fainter than $6^m{\cdot}0$. No determination was available for two stars in this group. The star which had moved out of the original diagram is placed below the group of fast-moving stars. The three stars next given also belong to the group, but as they are outside the limits of the region under examination they have not been included in taking means.

The next group contains the stars having proper motions about $1''{\cdot}5$ per century, together with additional stars which it is now possible to place in this group. The group now numbers 25, of which one has no colour determination, and one a weak determination (bracketed). The means of columns previously mentioned are also taken in this group.

A comparison of the parallaxes, proper motions, and C.E. of the two groups shows that the nearer fast-moving group is composed of stars with nearly normal colour, whilst the other group is made up of more distant stars with considerable C.E. The last column of the table gives, for comparison, the spectrophotometric gradient as determined at Greenwich (the gradient of an Ao star being 0·00). It therefore appears that the separation

* *Pub. Washburn Obs.*, **15**, 217, 1934.

by proper motion into two groups is well supported by colour determinations. Column 5 gives r, the distance in parsecs calculated by Stebbins by using the apparent magnitude and the mean absolute magnitude of the spectral type of the particular star corrected for colour excess. From the mean parallax and mean proper motions the distance of the two groups is 135 and 320 parsecs respectively. Pannekoek * gives values of 150 and 320 parsecs for the two groups. The stars in the nearer group (mean type B5) are, according to Stebbins, very slightly reddened to a mean type B6, the mean C.E. being $+0^m.03$, while the more distant group (mean type B2) is reddened to mean type A2 with a C.E. of $+0^m.14$. This means that in the 185 parsecs between the two groups the selective absorption amounts to $0^m.11$ or $0^m.58$ per kiloparsec. An independent value may be obtained from the Greenwich spectrophotometric gradients †. The mean C.E. from Stebbins for the

GROUP 1

(*near*)

Boss No.	Name or B.D. No.	Vis. Mag. H.D.	Spectral Type (Victoria)	r Stebbins	π Spect.	Proper Motion Schlesinger		C.E. Stebbins	G. Greenwich
				Parsecs	″	″	°	m	
740	30 Pers	5·38	B7*n*	130	·007	·040	136	+·04	
742	29 Pers	5·30	B3	210	5	47	133	+·01	
744	31 Pers	5·08	B3	190	6	43	132	+·03	
767	48° 899	5·30	B3	210	7	38	128	+·05	
780	48° 920	4·94	B5	130	8	33	129	+·07	
783	49° 944	5·64	B8*nn*	130	6	51	120	+·04	
790	34 Pers	4·67	B5	120	8	45	139	+·05	
796	47° 844	6·04	B9		8	48	137		
817	ψ Pers	4·26	B0*e*	290	8	45	127	+·08	
838	δ Pers	3·10	B8*n*	40	15	47	136	−·03	−·16
898	47° 912	5·34	B7*n*	120	6	41	128	+·02	
910	ϵ Pers	2·96	B2	50	6	41	137	·00	−·23
947	*c* Pers	4·03	B3*e*	110	13	45	133	+·05	+·06
1003	*d* Pers	4·89	B4*k*	140	6	46	147	+·05	+·01
1253	15 Caml	6·23	B5	230	4	36	142	+·07	
1274	ρ Auri	5·12	B7	120	6	46	144	−·02	
678	51° 665	5·42	B8*n*	110	11	45	130	+·03	
	48° 913	5·91	B8	170	5	30	135	+·01	
802	47° 847	5·52	B8*s*		6	41	137		
833	37° 811	5·57	B8*nn*	130	6	34	133	+·01	
				Mean	·0074	·042		+·03	−·08
1204	η Auri	3·28	B3	83	13	82	157	−·02	−·22
415	2 Pers	5·64	B9		8	40	141		
715	55 Aris	5·60	B9*n*		6	41	131		
1288	22 Auri	6·39	B9		6	40	138		

* *Pub. Astr. Inst. Amsterdam*, No. 2, 1929.

† *Observation of Colour Temperatures of Stars*, Vol. 2 (in preparation).

Greenwich stars agrees with the mean for the whole group, and so the two results are comparable. For 185 parsecs we find an absorption of 0·46 in gradient or 2·5 in gradient per kiloparsec. The photo-electric effective wave-lengths are λ 4250 and λ 4770, from which it may be deduced that a photo-electric colour index of $1^m.0$ corresponds to a gradient of 3·6, and therefore a selective absorption of $0^m.58$ per kiloparsec obtained from the photo-electric results corresponds to 2·1 in gradient per kiloparsec.

GROUP 2

(*more distant*)

Boss No.	Name or B.D. No.	Vis. Mag. H.D.	Spectral Type (Victoria)	r Stebbins	π Spect.	Proper Motion Schlesinger		C.E. Stebbins	G. Greenwich
				Parsecs	″	″	°	m	
	56° 470	7·21	B2*sk*	660	·001	·01	55	+·15	
507	56° 471	6·42	Bo*sk*	790	1	·016	97	+·18	
519	56° 522	6·66	B2*sk*	480	1	·016	135	+·25	
	56° 530	6·66	B1*sk*	580	1	·04	175	+·31	
529	46° 557	6·08	B7	190	4	·007	164	·00	
544	10 Pers	6·24	B1*sk*	500	1	·010	342	+·27	
741	65° 340	4·76	B3*ek*	170	5	·013	123	+·04	
781	59° 660	4·42	B9*p*		4	·004	117		
839	40 Pers	5·04	B2	210	3	·017	135	+·10	
844	*o* Pers	3·94	B2*k*	130	6	·028	154	+·12	+·33
	31° 649	6·51	B5*n*	220	3	·01	270	+·15	
	33° 717	6·36	B3	290	3	·02	208	+·17	
886	33° 728	5·73	B2	290	2	·013	94	+·12	
894	ζ Pers	2·91	B1*s*	120	7	·023	140	+·17	+·35
904	34° 768	5·48	B3	210	4	·019	234	+·07	
913	ξ Pers	4·05	O7*nk*	380	3	·020	131	+·09	+·21
992	50° 973	5·54	B3*n*	190	3	·014	82	+·07	
1050	1 Caml	5·86	B2*nk*	280	3	·008	135	+·12	
1139	*a* Caml	4·39	O9*sek*	440	3	·007	56	+·09	+·17
	35° 930	6·18	B2*sk*	360	3	·02	243	+·28	
1195	11 Caml	5·31	B3*e*	210	5	·008	194	+·05	
	37° 1067	6·17	B2*k*		3	·04	194	(+·41)	
1249	34° 980	5·81	O9*ssk*	790	3	·029	12	+·16	
	37° 1146	6·71	O6*sfk*	1200	1	·00		+·17	
1333	χ Auri	4·88	B3*ss*	170	4	·018	153	+·18	+·83
				Mean	·0031	·016		+·14	+·38

A great deal of investigation is now being carried out to obtain values for the general and selective absorption in different parts of the sky and results for the former deduced from star counts, and for the latter from colour measures already known are given below for the region under review.

Lehmann-Balanowskaja* has studied the region near ξ Persei ($l = 129°$; $b = -13°$) and finds that the cloud begins at 260 and ends at 500 parsecs. The general absorption is $2^m.1$ per kiloparsec and the selective absorption $0^m.9$

* *Pulkova Bull.*, 14, No. 1, 1935.

per kiloparsec using international colour indices. A more distant cloud in the same region commences at 1250 parsecs. Dyson and Melotte * give a region of obscuration near o Persei, the distance being 200–300 parsecs absorbing 2 magnitudes. Schalen † finds a cloud in Auriga at 125 parsecs absorbing $1^{m}.8$, while Greenstein ‡ gives a distance of 145 parsecs with a diameter of 65 parsecs absorbing $1^{m}.1$ for Tau-Ori-Aur cloud. Now our near group lies at 135 parsecs and appears to be only very slightly reddened. It must therefore be on the near side of, but probably just within, a cloud whose existence has been verified by several investigators. Our distant group is beyond this cloud and is reddened by it.

Summary.—The B-type stars in the Perseus-Auriga region are separated into two groups, near and more distant, by proper motions and parallaxes. The separation is confirmed by Stebbins's colour measures and by Greenwich spectrophotometric determinations. A selective absorption is derived and results are given of other investigations for both general and selective absorption in this region.

* *M.N.*, **80**, 3, 1919.

† *Kungl. Svenska Vetenskapakademiens Handligar*, Band **6**, 6: see also *Arkiv för Matematik*, Band **19** A, No. 33, 1926.

‡ *H.A.*, **105**, 359, 1937.

OCCULTATIONS OBSERVED AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE, IN 1938

(Communicated by H.M. Astronomer)

The occultations were reduced by the method and with the constants given in the *Nautical Almanac* for 1938, except No. 38 which was reduced in the same manner as in previous years (see *M.N.*, **96**, 808, 1936).

Observers :

Dr. J. Jackson	J.	Mr. A. Menzies	A.M.
Dr. R. H. Stoy	S.	Mr. J. B. Turner	J.T.
Mr. J. W. Jackson	J.J.	Mr. A. W. Long	L.
		Mr. W. P. Hirst	H.

Co-ordinates of instruments used :

Instrument	Longitude	Latitude
	h m s	° ′ ″
10-inch equatorial	1 13 54·70 E.	−33 56 7·8
18- ,, ,,	54·57	6·8
8- ,, ,,	54·57	6·8
7- ,, ,,	54·64	7·0
6- ,, ,,	54·69	7·1

For all these instruments we may take

$$\text{Log } \rho \cos \phi' = 9{\cdot}91936 \quad \text{and} \quad \text{Log } \rho \sin \phi' = 9{\cdot}74436n.$$

A correction of $+3''{\cdot}0$ has been added to the Moon's mean longitude as given in the *Nautical Almanac*, except for No. 46 for which $+1''{\cdot}5$ was used.

An analysis of the equations of condition gives the corrections to the orbital longitude ΔL and latitude ΔB :

Mean Epoch	ΔL	Wt.	ΔB	Wt.
	″		″	
1938·50	+1·99	29·5	−0·78	15·2

Two observations made in 1937 but not previously published are given.

No.	Date	Star	Mag.	Phase	Instrument	Obs.	U.T.
	1938						h m s
1	Mar. 9	B.D. +21° 902 (pr.)	7·8	D.D.	10-inch	J.	17 45 22·5
2	,, 9	,,	,,	,,	8- ,,	J.J.	17 45 22·5
3	,, 9	,,	,,	,,	18- ,,	J.T.	17 45 22·5
4	,, 9	,, (foll.)	7·2	D.D.	10- ,,	J.	17 45 32·2
5	,, 9	,,	,,	,,	8- ,,	J.J.	17 45 32·4
6	,, 9	,,	,,	,,	18- ,,	J.T.	17 45 32·4
7	,, 9	B.D. +21° 918	6·3	D.D.	10- ,,	J.	19 45 21·9
8	,, 20	41 Libræ	5·5	R.D.	7- ,,	S.	22 44 27·3

No.	Date	Star	Mag.	Phase	Instrument	Obs.	U.T.
	1938						h m s
9	Mar. 21	κ Libræ	5·0	R.D.	7-inch	S.	0 34 34·7
10	,, 25	B.D. − 18° 5206	6·4	R.D.	7- ,,	S.	0 38 19·0
11	Apr. 4	51 Tauri	5·6	D.D.	10- ,,	J.	17 26 46·1
12	,, 4	,,	,,	,,	7- ,,	S.	17 26 46·5
13	,, 4	53 Tauri	5·4	D.D.	10- ,,	J.	18 16 48·0
14	,, 4	,,	,,	,,	13- ,,	L.	18 16 48·4
15	,, 4	,,	,,	,,	7- ,,	S.	18 16 48·4
16	May 9	B.D. − 1° 2546	6·2	D.D.	10- ,,	S.	19 17 16·3
17	,, 9	,,	,,	,,	13- ,,	A.M.	19 17 16·5
18	June 5	B.D. − 0° 2442	6·3	D.D.	7- ,,	S.	21 9 3·3
19	,, 6	B.D. − 4° 3235	6·5	D.D.	7- ,,	S.	18 7 49·9
20	,, 6	,,	,,	,,	10- ,,	A.M.	18 7 50·0
21	,, 7	49 Virginis	5·3	D.D.	10- ,,	S.	21 48 16·0
22	July 22	54 Arietis	6·5	R.D.	7- ,,	S.	1 52 27·3
23	Aug. 3	B.D. − 17° 4273	7·0	D.D.	7- ,,	S.	21 53 48·4
24	,, 3	26 Libræ	6·3	D.D.	7- ,,	S.	23 5 32·3
25	,, 4	B.D. − 19° 4295	7·1	D.D.	7- ,,	S.	21 29 11·3
26	,, 4	β^1 Scorpii	2·9	D.D.	7- ,,	S.	22 46 29·8
27	,, 4	B.D. − 19° 4308	5·1	D.D.	7- ,,	S.	22 46 31·6
28	,, 5	B.D. − 20° 4572	5·9	D.D.	10- ,,	S.	20 46 53·8
29	,, 7	B.D. − 19° 5077	7·2	D.D.	7- ,,	S.	20 40 36·5
30	,, 8	ρ Sagittarii	4·0	D.D.	7- ,,	S.	17 34 8·1
31	,, 8	,,	,,	,,	10- ,,	A.M.	17 34 8·5
32	,, 18	π Arietis	5·4	R.D.	7- ,,	S.	0 12 12·2
33	,, 18	45 Arietis	5·9	R.D.	7- ,,	S.	4 9 12·0
34	Sept. 5	61 Sagittarii	5·0	D.D.	7- ,,	S.	19 55 58·5
35	,, 17	ζ Tauri	3·0	R.D.	10- ,,	S.	3 56 8·8
36	Oct. 1	B.D. − 18° 5079	6·5	D.D.	10- ,,	S.	17 50 8·1
37	,, 1	,,	,,	,,	18- ,,	J.T.	17 50 8·3
38	,, 1	B.D. − 18° 5080	7·0	D.D.	18- ,,	J.T.	17 52 26·0
39	,, 2	54 Sagittarii	5·4	D.D.	10- ,,	S.	19 42 53·4
40	,, 2	,,	,,	,,	6- ,,	H.	19 42 53·4
41	,, 2	55 Sagittarii	5·1	D.D.	10- ,,	S.	20 59 27·1
42	,, 2	,,	,,	,,	6- ,,	H.	20 59 27·0
43	Dec. 13	B.D. + 0° 2782	6·3	R.D.	7- ,,	S.	23 35 18·5
44	,, 16	B.D. − 8° 3491	5·7	R.D.	7- ,,	S.	0 54 54·6
45	,, 16	B.D. − 8° 3495	6·4	R.D.	7- ,,	S.	1 40 58·8
46	,, 28	B.D. + 1° 4731	7·1	D.D.	10- ,,	J.	20 0 18·6

Equations of Condition

1–3	$+3''\cdot0$	$= +13\cdot0\Delta\alpha$	$- \cdot36\Delta\delta$	$= + \cdot94\Delta L$	$- \cdot33\Delta B$
4–6	$-0\cdot5$	$= +13\cdot0$	$- \cdot36$	$= + \cdot94$	$- \cdot33$
7	$+0\cdot3$	$= + 8\cdot1$	$- \cdot81$	$= + \cdot62$	$- \cdot79$
8	$+2\cdot4$	$= - 14\cdot2$	$+ \cdot03$	$= - \cdot99$	$- \cdot12$
9	$+2\cdot0$	$= - 11\cdot2$	$- \cdot61$	$= - \cdot70$	$- \cdot72$
10	$+1\cdot2$	$= - 3\cdot8$	$- \cdot97$	$= - \cdot42$	$- \cdot91$
11–12	$-1\cdot2$	$= +11\cdot1$	$+ \cdot60$	$= + \cdot85$	$+ \cdot53$
13–15	$-1\cdot4$	$= + 3\cdot6$	$- \cdot96$	$= + \cdot18$	$- \cdot98$
16–17	$+0\cdot1$	$= + 5\cdot9$	$- \cdot92$	$= + \cdot69$	$- \cdot72$
18	$+1\cdot4$	$= + 1\cdot1$	$-1\cdot00$	$= + \cdot42$	$- \cdot91$
19–20	$-1\cdot3$	$= +11\cdot8$	$- \cdot61$	$= + \cdot95$	$- \cdot30$

	″					
21	+1·7	= + 1·8	− ·99	= + ·42	−·91
22	+1·6	= −13·7	+ ·27	= − ·89	+·45
23	−0·3	= +11·3	− ·61	= + ·89	−·46
24	−1·2	= + 8·2	+ ·82	= + ·42	+·91
25	+1·1	= +12·3	− ·49	= + ·92	−·39
26	−1·5	= +13·0	− ·38	= + ·96	−·28
27	−2·1	= +13·1	− ·36	= + ·97	−·26
28	−1·7	= +10·1	+ ·69	= + ·70	+·72
29	−1·1	= + 7·6	+ ·84	= + ·63	+·78
30–31	−1·6	= +13·7	− ·28	= + ·89	−·45
32	−0·5	= −13·3	− ·36	= − ·99	−·15
33	+1·0	= − 9·1	+ ·77	= − ·46	+·89
34	−1·6	= + 8·7	+ ·79	= + ·77	+·64
35	+2·1	= −13·2	− ·35	= − ·93	−·37
36–37	−1·3	= +14·1	+ ·13	= +1·00	−·01
38	−2·8	= +14·2	− ·01	= + ·99	−·14
39–40	−1·5	= +14·0	+ ·22	= +1·00	+·02
41–42	−1·2	= +14·1	+ ·19	= +1·00	−·02
43	+2·2	= −12·6	+ ·55	= − ·98	+·22
44	−1·0	= − 1·8	+ ·99	= − ·42	+·91
45	+1·8	= −12·4	− ·55	= − ·62	−·78
46	−1·4*	= + 2·2	+ ·99	= + ·48	+·88

1–3 and 4–6. Two components 3″·6, 262°.
8. Observation doubtful.
22. ,, ,,
23. Seeing very poor.
24. ,, ,, ,,
25. Disappearance sluggish.
26. Definition very poor.
27. ,, ,, ,,
34. Cloud passing.
38. App. place from *Albany General Catalogue*, $18^h\ 45^m\ 14^s{\cdot}29$, −18° 41′ 57″·7.
43. Appearance sluggish.
46. Through cloud. Correction of +1″·5 to Moon's mean longitude used.

No.	Date	Star	Mag.	Phase	Instrument	Obs.	U.T.
	1937						h m s
1	Aug. 16	B.D. −21° 4682	6·7	D.D.	10-inch	S.	22 55 49·5
2	,, 19	B.D. −18° 5356	7·0	D.D.	10- ,,	S.	0 17 57·3

Aug. 16. Seeing very poor.
,, 19. Moon bright. Star very difficult to follow.

Equations of Condition

	″				
1	−0·4 = +13·5Δα	+·23Δδ	= +·98ΔL	+·18ΔB
2	−1·8 = +13·6	−·27	= +·88	−·47

OCCULTATIONS OF STARS BY THE MOON OBSERVED AT BIRMINGHAM UNIVERSITY DURING THE YEAR 1938

James Young

This report continues the series begun in 1928.

In Table I the U.T.'s are uncorrected for lag. The observations were all made under good conditions. Nos. 210, 212, 213 and 215 were independently observed by J. V. Jelley.

The reductions are given in Table II. The quantities a and b are the coefficients of $\Delta\alpha$ and $\Delta\delta$ in the expression for correcting $\Delta\sigma$ for an alteration in the adopted position of the star. A correction of $0^h{\cdot}00152$ was made to the U.T.'s before interpolating the quantities x, y, F and G from the values given in the *Nautical Almanac*. These reductions were independently checked by G. S. Pearce.

Table III gives the positions of stars for which predictions were not made in the *Nautical Almanac*.

TABLE I

No.	Star	Mag.	Phase	Date	U.T.
				d	h m s
191	19 Arietis	6·0	*D*	Jan. 10	19 38 45·2
192	B.D. +20° 785	5·7	*D*	Mar. 8	18 48 10·5
193	B.D. +20° 802	9·0	*D*	Mar. 8	20 41 31·6
194	B.D. +16° 1551	7·4	*D*	Mar. 11	22 04 44·2
195	A^1 Cancri	5·7	*D*	Mar. 12	21 49 48·7
196	A^2 Cancri	5·7	*D*	Mar. 12	24 07 09·8
197	B.D. +18° 484	6·4	*D*	Apr. 3	21 10 45·0
198	B.D. +20° 961	7·7	*D*	Apr. 5	20 55 28·1
199	B.D. +20° 969	6·8	*D*	Apr. 5	21 31 59·3
200	B.D. +20° 974	8·4	*D*	Apr. 5	22 00 01·8
201	19 Sextantis	5·9	*D*	Apr. 10	20 36 30·2
202	62 Leonis	6·2	*D*	Apr. 11	19 50 12·9
203	B.D. +15° 1775	6·1	*D*	May 5	20 14 10·5
204	Anon	9	*D*	May 5	21 17 07·2
205	B.D. +14° 1850	6·4	*D*	May 5	21 27 25·1
206	B.D. +1° 2495	6·3	*D*	May 8	20 52 29·3
207	87 Leonis	5·1	*D*	June 5	21 29 29·7
208	29 Arietis	6·1	*R*	Sept. 13	22 28 44·9
209	λ Piscium	4·6	*D*	Oct. 7	21 08 50·3
210	B.D. −9° 5827	7·1	*D*	Nov. 1	18 26 12·0
211	B.D. +15° 407	7·8	*D*	Nov. 7	22 43 59·4
212	B.D. −10° 5696	7·4	*D*	Nov. 28	20 42 19·1
213	B.D. −7° 5727	7·4	*D*	Nov. 29	17 15 52·3
214	B.D. −3° 5539	6·2	*D*	Nov. 30	16 53 55·6
215	22 Piscium	5·8	*D*	Dec. 1	21 49 30·8
216	B.D. +9° 167	7·2	*D*	Dec. 3	17 45 35·3

TABLE II

No.	Lun.	p	q	p^2	pq	q^2	$\Delta\sigma$	$p\Delta\sigma$	$q\Delta\sigma$	a	b
							″	″	″		
191	186	+ 49	−87	24	−43	76	−0·8	−0·4	+0·7	+10	−0·7
192	188	+100	+ 4	100	+ 4	0	+0·5	+0·5	0·0	+14	+0·1
193	188	+ 49	+87	24	+43	76	−0·3	−0·1	−0·3	+ 6	+0·9
194	188	+ 96	+29	92	+28	8	−0·6	−0·6	−0·2	+14	+0·1
195	188	+ 99	+15	98	+15	2	−0·2	−0·2	0·0	+15	−0·1
196	188	+ 91	−41	83	−37	17	−0·4	−0·4	+0·2	+11	−0·7
197	189	+ 61	+79	37	+48	63	−0·7	−0·4	−0·6	+ 7	+0·9
198	189	+100	− 8	99	− 8	1	+0·9	+0·9	−0·1	+14	−0·1
199	189	+ 88	+48	77	+42	23	−0·8	−0·7	−0·4	+13	+0·5
200	189	+ 67	−74	45	−50	55	0·0	0·0	0·0	+ 9	−0·8
201	189	+ 51	−86	26	−44	74	−0·4	−0·2	+0·3	+ 3	−1·0
202	189	+ 98	−22	95	−22	5	−0·8	−0·8	+0·2	+13	−0·6
203	190	+ 73	+69	53	+50	47	0·0	0·0	0·0	+13	+0·5
205	190	+ 23	−97	5	−22	95	0·0	0·0	0·0	0	−1·0
206	190	+ 91	+42	82	+38	18	−1·6	−1·5	−0·7	+15	+0·1
207	191	+ 87	−49	76	−43	24	+0·5	+0·4	−0·2	+10	−0·8
208	194	− 99	−16	97	+16	3	−0·6	+0·6	+0·1	−13	−0·4
209	195	+ 97	+26	93	+25	7	−1·7	−1·6	−0·4	+12	+0·6
210	196	+ 64	−77	41	−49	59	−0·9	−0·6	+0·7	+13	−0·5
211	196	+100	− 1	100	− 1	0	−1·7	−1·7	0·0	+14	+0·2
212	197	+100	+ 1	100	+ 1	0	−1·2	−1·2	0·0	+14	+0·3
213	197	+ 99	−14	98	−14	2	−1·3	−1·3	+0·2	+15	+0·2
214	197	+ 97	+25	94	+24	6	−4·1	−4·0	−1·0	+15	+0·1
215	197	+ 97	+23	95	+22	5	−1·8	−1·7	−0·4	+13	+0·6
216	197	+ 79	−61	63	−48	37	−1·5	−1·2	+0·9	+14	−0·4

TABLE III

No.	α 1938·0	$\Delta\alpha$	δ 1938·0	$\Delta\delta$	Authority
	h m s	s	° ′ ″	″	
193	4 37 39·64	+1·64	+20 47 13·5	− 1·0	Yale
198	5 25 16·84	+1·39	+20 23 34·7	− 3·9	,,
200	5 27 24·31	+1·40	+20 08 34·7	− 4·1	,,
211	2 51 49·42	+5·06	+16 03 32·7	+17·5	Boss *G.C.*

The University Observatory,
Edgbaston, Birmingham :
1939 *March* 6.